AN ECONOMIC ANALYSIS OF UNITED STATES WHEAT CARRYOVER POLICIES

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PREFACE

This dissertation is concerned with investigating United States wheat reserve stock management policies. The stocking activity is viewed from an aggregate level, and begins with the assumption that national reserve stocks are desirable. The analysis is divided into two separate but interrelated parts. The first part is concerned with determining an optimal level of wheat carryover while the second part deals with investigating the effects on the wheat economy of various reserve stock management policies.

I wish to express my sincere appreciation to Dr. Luther G. Tweeten for his guidance and encouragement as chairman of my graduate advisory committee and director of this research effort. Appreciation is also expressed to all members of my advisory committee -- Dr. Leo V. Blakley, Dr. Frank G. Steindl and Dr. David Weeks -- for their support and instruction throughout my graduate program.

Special acknowledgement is due Dr. Yao C. Lu who demonstrated that the dynamic programming approach is adaptable to the wheat carryover problem and who helped me understand the dynamic programming technique. Dr. Lu also wrote the basic computer programs which were used to prepare much of the input data for the dynamic programming analysis.

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CHAPTER I

INTRODUCTION

Stabilization of farm prices and incomes has been a major objective of commodity programs for four decades. To achieve this objective, the government has used means including direct payments, production controls and, to a lesser extent, commodity storage. The "stabilization" function has had at least two dimensions for farm prices and incomes: raising their level and reducing their variability. With growing demands for public funds for urban and other national problems and realization that the benefits of higher levels of farm prices and incomes are lost through capitalization of benefits into farmland, increasing attention is focusing on the second dimension of commodity programs: reducing the variability in farm prices and incomes. An important issue is whether carefully formulated commodity storage policies, either alone or in combination with other measures such as production control, can reduce the variability of farm prices and incomes to reasonable levels at low Treasury cost.

The purpose of this study is to provide information that will help policy makers make better decisions about the management of reserve stocks of wheat. The model developed here and the empirical analyses apply directly only to wheat, but could be extended, with some modifications, to other storable agricultural commodities.

A number of questions can be raised regarding the many relationships between holding reserve stocks and other policy issues, but many of these are outside the bounds of this study. We begin by taking as given that: 1) some level of stocks are desired by society, and 2) society is willing to pay a certain price for stock acquisition and management. Not to be considered directly are issues such as: 1) where stocks should be held and by whom, 2) incidence and distribution of costs and benefits of a reserve stock program, and 3) relationships between reserve stock programs and other farm programs. Interest will center instead on attempting to determine how large the reserve stocks should be and on how various reserve management policies affect certain key economic variables.

The Nature of the Reserve Management Problem

Uncertainties associated with wheat reserve stock management may be either in the supply of, or in the demand for the grain, or both. On the demand side, the major source of variation is in the demand for export: domestic demands change very little from year-to-year, but export demand varies widely as production in the rest of the world fluctuates and as economic and military assistance programs change.

On the supply side, variation arises from two sources: yield per acre and acreage planted. Government farm programs of recent years have demonstrated their ability to influence acreage planted to the extent that this can be considered a policy variable and not a random variable. Yield per acre is subject to considerable random variation -- mainly due to the effects of uncertain weather conditions.

If quantities are not sometimes stored from one year to the next, the interaction of variations in exports on the demand side and in yields on the supply side have potentially undesirable effects. Supplies that are very small relative to demands result in high prices and/or short supplies -- undesirable to consumers, whether domestic or foreign. Undesirable to farmers are the instabilities caused by wide fluctuations in prices and incomes that unchecked supply and demand can bring.

Several authors have listed or discussed various goals or objectives of a national reserve policy. These goals can usually be fitted into three categories.

1. To prevent the shortages that might result from adverse production conditions. The term shortages may be broadly or narrowly interpreted, depending on whether or not one wishes to include producing areas outside the United States.
2. To provide a degree of economic stability for the agricultural and nonagricultural sectors of the economy. Stability would be induced by buying and selling operations which help prevent the high prices associated with shortages or near shortages and the low prices that would normally result from a glutted market during years of greater-than-normal supplies.
3. To provide stocks that can be used for military and economic aid to threatened and/or developing nations.

Each of these objectives involves or provides some form of protection to a certain group or groups. The exact form taken by a national reserve policy depends on how much protection the public is willing to provide these groups and on how the various objectives are ordered or weighted. Or put another way, a criterion or objective function is needed which will measure the utility of having reserve stocks -- a way of evaluating the trade-off between protection and cost.

If an objective measure of the value of a reserve stock program can be formed and policies developed according to the results obtained by optimizing such a measure, several benefits should be forthcoming. With an established policy, the tendency to use farm programs for political gain might not be so great. Producers like to see situations where commodities are taken off the market, but don't like for them to be released. If reserve management rules could be soundly developed from an optimizing procedure and well established with present guidelines so that farmers always know what the rules are, there might be less distrust among farmers toward inventories being held by government and less chance that the program would be subject to changes made for political advantage.

Another need is that even with some knowledge of the uncertainty surrounding supply and demand, and with an acceptable measure of the public benefits from having reserves, a way must be found that will determine the level of carryover to optimize this measure of social utility. To be a practical aid to policy making, the utility measure must be able to use available data and the optimizing procedure must be fairly easy to use.

Still another need associated with reserve stock management is to be able to evaluate the effects of various reserve management strategies on certain economic variables such as farm prices and incomes, government costs, availabilities of supplies, etc. The important policy issue here is ability to determine which, if any, of the important economic variables are sensitive to inventory strategies that are nonoptimum. This would aid in evaluating rule-of-thumb and other reserve management policies.

Objectives of the Study

As stated earlier, the purpose of this study is to provide information of value to policy makers. To accomplish this overall goal, it is necessary to attain certain methodological and empirical objectives. The methodological goals are necessary in order to provide a framework or an analytic procedure which may be used to analyze the problem as described in the previous section. Attaining the empirical objectives will provide, among other things, estimates of relevant economic variables, their interactions, and their reactions to manipulation of other policy variables.

Methodological Objectives

The overall methodological objective of this study is to develop techniques which will provide answers to, or at least shed light upon, the United States wheat inventory problem. Toward this end, three major methodological issues may be stated as objectives.

1. To develop a mathematical statement as a measure of the benefits to society resulting from year-to-year carryover of wheat.
2. To select an optimizing procedure adaptable to the wheat inventory problem which will provide an optimum value for the measure of benefits developed in (1) above.

3. To develop an aggregate model of the United States wheat economy suitable for use as a vehicle to examine the efficiency and effectiveness of techniques and concepts of (1) and (2) above and of alternative policies applied to the wheat inventory management problem.

Empirical Objectives

Specific empirical objectives of this study are:

1. To determine the optimum carryover of stocks of wheat under specified market conditions.
2. To evaluate the expected results from implementing management policies dictated by the optimization procedure compared with other current or foreseeable reserve policies. This comparison will focus on the level, stability, and variability of relevant economic variables such as wheat price, farm income, and the adopted measure of social benefits.

Procedures Used in the Study

The relationships between the empirical and methodological objectives are conditioned by one another and by outside influences. The procedures used to analyze a problem obviously place restrictions on the types of answers or results that are forthcoming, and similarly, the particular answers desired limit the analytic techniques which may be used. External forces such as limited time, limited research money, availability (or nonavailability) and quality of data also affect the choice of research technique and the final results. These considerations, together with the inherent nature of the wheat reserve-stock problem, have influenced the choice of procedures used in this study.

Optimization Technique -- Dynamic Programming

For any period, an optimum storage level implies an optimum allocation of supply between this period and succeeding periods. For an optimum level to be identified, some judgment must be made about the value (or cost) to society of placing various quantities into storage (increasing future years' supplies) versus placing these quantities on the market. It is only by placing such a value on each storage alternative that an alternative can be chosen which optimizes this value. For this study, the effectiveness or objective function chosen to assign these values is a measure of social cost which will be explained in Chapter II.

The decision as to how much of an available supply should be used in future years depends upon how the quantity of carryover is to be distributed among all future years. But this year's supply is made up of this year's production and carryover from last year -- a result of a decision made last period based on the level of supply last period. Thus, the optimal decision to be made in any period is a function of both the total supply available and the decision made in the immediate and all other preceding periods. The optimizing model then, must be a multistage model, capable of optimizing decisions made sequentially or serially.

Another complication arises from the uncertain nature of any year's production. The optimal decision concerning this year's carryover is dependent upon the carryover decision of last period which was made before this year's production was known. So in addition to a sequential model, the optimization procedure must be able to deal with a stochastic variable.

The system under consideration then, may be classified as a serial, multistage, stochastic system. A technique of analysis that lends itself to this type of model is dynamic programming. The dynamic programming technique will be discussed in more detail in Chapters II and III: sufficient now to say that dynamic programming is a name for an approach to sequential decision making or recursive optimization. The approach is suited to either deterministic or stochastic models. The optimization procedure then, is to use stochastic dynamic programming to minimize the sum of carryover and social costs over time.

Simulation

A stochastic computer simulation model has been developed to evaluate the effectiveness of various inventory policies. Simulation is not generally an optimizing tool, but a properly constructed model allows the researcher to evaluate the effects of changes in parameters and assumptions of the model via relatively simple operations upon the model itself. The simulation model developed for this project is of the Monte Carlo nature and contains built-in stochastic processes to approximate uncertain yield and export demand. Various runs of the model generate statistics with which to compare different inventory policies and market characteristics.

Review of Literature

Grain Inventory Management Analyses

Attention has been directed toward the grain inventory management problem -- or at least toward determining satisfactory reserve levels -- in relatively few economic analyses. A 1952 study by Karl Fox and O. V. Wells (1) for the Senate Agricultural and Forestry Subcommittee,

> . . . endeavors to analyze yield and other data in order to indicate the stocks or reserve levels which will be necessary to offset specified yield variations for three of the main storable crops -- corn, cotton, and wheat -- and to state the several leading policy questions which need to be considered by farmers, the Congress, and public officials in arriving at a final determination as to what stocks or reserve levels seem most desirable or feasible, including the determination of the conditions under which such stocks might be carried and released. (1, p. 1).

The approach of this study was essentially to analyze historical yield and consumption variation to determine probabilities associated with various production deficits over a two-year period. It was determined that a reasonable storage objective would be to stock sufficient quantities to offset one year of very low yield (an adjusted average of the five lowest yields) followed by one year with a moderately low yield (average of the next 20 lowest yields). They then concluded that reserves of 500 million bushels of wheat, 1 billion bushels of corn and 5 million bales of cotton would provide reasonably good protection.

Robert L. Gustafson (2) in 1958 conducted a study that contributed substantially to knowledge of the grain inventory management problem. A procedure was developed to maximize over time a measure of public benefits, namely total value or the area under the grain demand curve. Gustafson developed an optimal set of storage rules (a policy) which tells, for each possible supply and for several different market conditions and storage costs, the quantity to place into storage to maximize the expected public gain. Explicit account is taken of the stochastic nature of output and of the intertemporal dependence of supplies and decisions. The theory is developed for mathematical solutions: application of the technique to feed grains is accomplished by approximate numerical and graphical methods.

Fredrick V. Waugh (3) in a study conducted for the National Food and Fiber Commission in 1967 concludes that satisfactory goals for storable farm products are as follows:

Wheat	550-650 million bu.
Corn	.8-1.0 billion bu.
Four feed grains	35-40 million tons
Rice	10-12 million cwt.
Cotton	5-6 million bales.

Waugh's study extended the analysis of Fox and Wells by considering probable future variations in demand and substitution possibilities more explicitly. The quantitative analysis culminates in graphical relationships between percentages of years for which production has been less than percentages of trend. Because these curves are developed from production frequencies, they suggest a below-trend production level (or combination of two years below trend) against which it would be wise to be protected by reserves, and hence percentages of a normal crop necessary to achieve this protection. These values are adjusted for substitution possibilities (suggested by simple correlations), likelihoods of two or more small crops in sequence, and demand-variation considerations (suggested by correlations between yearly changes in crop production in certain pairs of countries). Also discussed are various aims or objectives of storage programs, their relationships to other policy considerations and past programs.

Vernon R. McMinimy and Francis A. Kutish (4) also discuss a reserve program for wheat and feed grains, pointing out possible objectives of such a program and the factors which should be considered in any model which attempts to determine United States reserve levels.

Dynamic Programming

Although dynamic programming is a relatively new research tool, there is already an imposing list of theoretical developments and empirical applications. Richard Bellman (5) is generally given credit for setting out the conceptual framework and mathematical treatment of multistage decision processes, as well as naming the approach "dynamic programming," in his 1957 book, Dynamic Programming. He has continued to publish books and articles dealing both with theoretical extensions and with applications to a wide variety of problems. Rather than attempt to chronicle or classify the literature here, reference is made to the fairly exhaustive bibliographies given by Bellman and Karush (6), Nemhauser (7), Scarf (8) and Kaufmann and Cruon (9).

Applications of the technique to problems of direct interest to agriculturalists have been relatively limited. The most notable exceptions are the contributions of Oscar R. Burt (10, 11, 12, 13). Burt has applied the dynamic programming technique of sequential decision theory to maximize a measure of value of output from groundwater basins. In this case, the output from the stochastic model is a policy to govern withdrawal of water from the basin over time (10). In another article, Burt and J. R. Allison (11) demonstrate the potential application of dynamic programming to farm management decisions by formulating a model using soil moisture at wheat planting time as a decision variable: the optimal policy dictates a wheat-fallow decision based on the soil moisture level at planting time.

Simulation

Because simulation is a very general and a very adaptable research tool, it has been used to study a wide variety of problems in many seemingly unrelated disciplines; consequently, the volume of literature is quite imposing. Reference will be made here to only a few examples which indicate simulation's versatility as a tool for economic analysis of agriculturally oriented problems. All of the studies cited here contain several references to other simulation literature.

A. N. Halter and G. W. Dean (14) have developed a simulation model to represent the decision making processes associated with the operation of a California range-feedlot operation. Uncertain weather and prices are incorporated into the model as stochastic variables: alternative buying policies (decisions) are then examined in the face of these uncertainties.

E. M. Babb and C. E. French (15, 16) report on the development of a stochastic simulation model for a particular Indiana cheese plant. Alternative milk buying and selling policies are simulated to develop that policy which provides the greatest net profit. Output from the model also has implications for production control and labor policies.

An intensive report on a simulated dairy herd operation is given by R. F. Hutton (17). Alternative policies of herd replacements are examined -- whether to buy or to raise replacements -- as they affect net profits. Several milk price and feeding program variations are considered as to effects on the choice of replacement policies.

Pinhas Zusman and Amotz Amiad (18) show how to use the simulation technique together with a steepest ascent procedure to give optimal farm management decisions under uncertainty. The simulation model includes provisions for a stochastic rainfall variable which determines crop seedbed conditions, forage inventory levels and natural forage available. The major decision variables deal with the allocation of acres to various crops. A response surface is generated by defining optimal decision rules in terms of the present value and coefficient of variation of net incomes. A search strategy is developed to search the response surface by the steepest ascent procedure to arrive at an optimal policy.

Organization of the Study

Chapter II of this study is concerned with the procedures used to develop an optimum wheat carryover policy. The wheat inventory problem is formulated as a decision model adaptable to the dynamic programming technique, and the dynamic programming procedure is outlined. The utility measure used as a policy guide is also presented.

Chapter III presents the results of the dynamic programming analysis. The input data necessary for the dynamic programming technique is given first, followed by the results of the optimization procedure and the implications of the results for policy decisions.

Chapter IV establishes the simulation models used in this study to generalize and test various reserve management policies including that developed from the results of Chapter III. General characteristics and features of the simulation approach are given. The operation of each model is explained with reference to inventory adjustments and equilibrium conditions.

Chapters V and VI present the results of the simulation analysis. Chapter V contains the results of the simulation of three inventory management models developed in Chapter IV and compares the results obtained from the simulation of each model. Chapter VI presents the results of various changes that were made within the model and the reasons for selecting these changes for examination. These chapters also give implications of the results of the simulation analysis for policy decisions.

Chapter VII presents a summary and the conclusions derived from this analysis. Recommendations for additional research are given also.

CHAPTER II

DYNAMIC PROGRAMMING FORMULATION OF A WHEAT INVENTORY MODEL

In this chapter, a decision model of the inventory management problem is formulated in such a way that the dynamic programming optimization technique is applicable. The dynamic programming method and characteristics of the results of the technique are briefly outlined. A measure of the value to society provided by reserve stocks is presented and explained in terms of the wheat inventory model. This measure will be used in the next chapter to determine empirically how much wheat should be put into storage each year and how to allocate this storage among future years' supplies.

The Social Cost and Total Loss Functions

The model requires that a value to society be placed on the quantities stored. Only in this way can storage alternatives be chosen to maximize that value. Luther Tweeten and Fred Tyner (19) have developed the utility concept of net social cost which will be used as the policy criterion for this study. This concept states that the net social cost from failure to utilize those quantities which exactly correspond to the economic equilibrium is given by the area bounded by the demand and supply curves and by the deviation of quantities actually utilized from the equilibrium quantity. This cost (or benefit foregone) is the difference between total utility gained (or total social benefit) as measured by the area under the market demand curve and total utility foregone as measured by the area under the market supply curve. As stated by Tweeten and Tyner:

> At any given wheat quantity, the vertical distance from the quantity axis to the demand curve is one measure of the social benefits of that quantity, and the distance to the supply curve is one measure of the social cost. It follows that the difference between these vertical segments, the distance between the demand and supply curves, is one measure of the net social gain from producing and consuming that particular quantity of (say) wheat. If we sum the net social gains for each bushel of wheat, the area between the supply and demand curves is traced. (19, p. 34)

For the wheat carryover problem, it is assumed that if the total supply available (this year's production plus carryover from last year) is less than or equal to an assumed equilibrium quantity, there will be no carryover. In this case the area between the demand and supply curves and bounded on the left by the quantity actually supplied and on the right by the equilibrium quantity is a measure of net social benefits foregone. This is shown for one period as the shaded area ABC in Figure 1 where Q^* is the equilibrium quantity and S' is the total available supply and also the quantity actually utilized.

If the supply available is greater than the equilibrium quantity, it is assumed that carryover will be positive. A measure of social cost is then given by the area between the supply and demand curves bounded on the left by the quantity actually utilized -- supply less carryover. In Figure 1, S is the total available supply, C is the carryover into the next period and Q is the quantity actually utilized so that net social cost is given by the shaded area CDE and is a decreasing function of the level of carryover. Also in this case, an additional cost is incurred in the form of storage cost which is an increasing function of the level of carryover.

If carryover costs were zero, social cost minimization would simply require that carryover be sufficient to cause Q^* to be used each period, or if supply were not random, there would be no need for carryover (assuming the demand and supply schedules were known with certainty). But positive carryover costs and random yields combined give minimum total social loss when carryover is such that the quantity actually utilized is greater than Q^* and less than S. For any one period then, the loss is either the net social benefit foregone or the net social cost plus the storage cost. The total loss for the planning period is the sum of the losses for each year in the planning period.

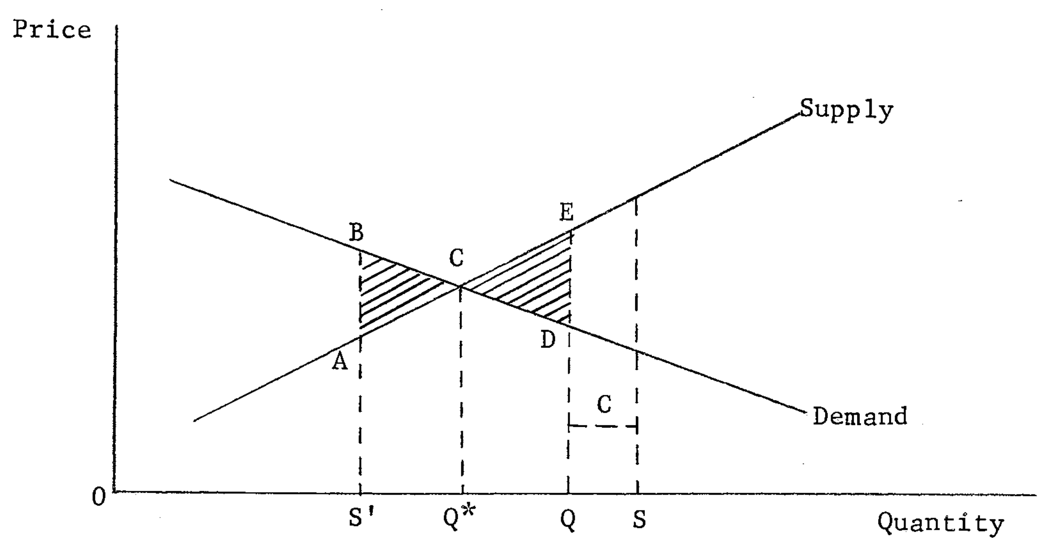

Figure 1. Illustration of Net Social Cost and Net Social Benefit Foregone

For each period there are two supply curves to consider: the short-run perfectly inelastic supply that is assumed stochastic due to the effects of random yields, and the long-run market or planning supply curve. The long-run supply curve is not perfectly inelastic because before each year's planting decision is made, planned production may be varied in response to price changes. After the planting decision is made, it is assumed that production will not respond to price changes, but varies only due to random yields and therefore is a vertical line.

It is also assumed that the long-run or planning supply curve is constant from period-to-period so that the area measured by a given deviation from equilibrium is the same for all periods. The same result could be obtained by assuming dynamic demand and supply functions which move together in such a way that any given deviation gives the same social cost for all periods.

If:

S_t is the total supply available in period t,

C_t is the carryover from period t to period t+1,

Q_t is the quantity utilized in period t,

Q^* is the equilibrium quantity,

P_D is the market demand function,

P_S is the market supply function, and

R(C) is the storage cost function,

then for any period t, the expected net loss is:

$$L(S,C) = \begin{cases} \int_{S_t}^{Q^*} (P_D - P_S)dq & , S_t \leq Q^* \\ R(C) + \int_{Q^*}^{S_t - C_t} (P_S - P_D)dq & , S_t > Q^*. \end{cases} \tag{1}$$

The economic criterion for optimal carryover is to choose a strategy or conditional decision rule that minimizes the expected value of discounted net losses over the planning horizon. This decision rule is a schedule of the quantities to store from one period to the next given a specific level of total supply. Associated with the optimal strategy is a function defined as the expected discounted net losses over the planning horizon when the optimal policy is followed. The objective is to arrive at these two functions.

The Stochastic Inventory Decision Model

To formulate the wheat carryover problem into a serial multistage decision model, the following additional definitions and notations will be used:

X_t, production in period (year) t;

Q_t, quantity utilized in year t;

C(S), a schedule relating total supply and carryover;

$C^*(S)$, the optimal schedule or strategy;

$V^*(S)$, expected discounted net losses over the planning horizon when $C^*(S)$ is followed;

L(S,C), net loss per period;

h(X), the distribution of production probabilities;

$B = (1 + r)^{-1}$, r is the discount rate per period.

Then,

$$Q_t = \begin{cases} S_t & S_t \leq Q^* \\ S_t - C_t, & S_t > Q^* \end{cases} .$$

Description of the Serial Multistage System

A serial multistage system consists of a series of stages joined together so that the output of one stage becomes the input to the next stage (7, p. 26). For our purposes, a stage will be one year in length and the n^{th} stage in the decision process is defined as a situation in which N-n years remain in the planning horizon of N years. The system is described at each stage by an input state variable S_t, the level of wheat supply at the beginning of the stage. The stages are connected by a stage transformation, T(S,C), which transforms the input state variable into an output state variable, S_{t+1}, the input state variable and level of supply for the next period. In other words, the state of the system at stage n is transformed into a new state at stage n+1 by the transformation function T(S,C) which expresses the components of the output state as a function of the input state and a decision variable. The variable C_t is the decision variable and is the amount of wheat to carry over to the next year.

Associated with each stage transformation is a loss measured by the stage return function L(S,C). For each possible input state S_t, C_t is to be chosen to minimize the total losses for the N periods (stages) in the planning horizon. T(S,C) is a random variable since one of its components, X_t, is stochastic with probability density function h(X).

Because supply in period t is a function of the random variable X_t and C_{t-1}, the decision variable of the previous period, we cannot determine the quantity of wheat to carry over (or to be used) for each year in the future, but must find instead the optimum level of carryover for each possible level of supply. Actual supply is a random variable, and before a proper (optimum) choice can be made concerning distribution of supply between use and carryover, we must wait to see what is the actual level of S_t. To apply the economic criterion for optimal carryover, it is necessary to find C_t, $t = 1, 2, \ldots, N$, to minimize the discounted expected total loss for years (stages) 1 through N.

A transformation of the variables X, S, and C into discrete variables will reduce the problem to a finite Markov decision process that allows the problem to be decomposed into N subproblems. The optimal decisions may be found one at a time, then combined to obtain the optimal solution to the original N-stage problem.

Define M discrete supply levels S_i, $i = 1, 2, \ldots, M$, each level representing a state, and K carryover levels C_k, $k = 1, 2, \ldots, K$. $L(S,C)$ may then be represented by L_i^k, the loss for input state i when alternative carryover k is chosen. Define p_{ij}^k to be the (transition) probability of state i being transformed into state j when alternative k is chosen.

The n^{th} stage return depends on the random variables $X_1, X_2, \ldots, X_{n-1}$ as well as X_n since X_n depends on the previously observed random variables $X_1, X_2, \ldots, X_{n-1}$, as well as the decision variables $C_1, C_2, \ldots, C_{n-1}$. Since any S_i depends only on the values of decision and random variables of previous stages and the current value of the random variable which results with probability P_i, the expected value of the total loss for n stages may be represented by $V_i(n) = L(S_1, C_1, C_2, \ldots, C_{n-1}, P_1, P_2, \ldots, P_n)$. This means that the total discounted series of losses of all N stages is determined by the initial supply, previous decisions about carryover, and the random variables (20, p. 6).

The Optimization Principle

The problem now becomes one of minimizing the N-stage losses over the decision variables $C_1, C_2, \ldots, C_N$. Bellman's principle of optimality provides the basis for formulating a recursive optimization equation: "an optimal policy has the property that whatever the initial state and initial decision, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision" (21, p. 57).

Define $f_i(n)$ as discounted expected loss during the last N-n years of the planning horizon under an <u>optimal policy</u>. For any choice of C at stage n, the expected discounted loss is:

$$V_i^k(n) = L_i^k + B \cdot \sum_{j=1}^{M} p_{ij}^k \cdot f_j(n+1) \quad i = 1, 2, \ldots, M \tag{2}$$

where

$$f_i(n) = \operatorname*{Min}_k V_i^k(n). \tag{3}$$

This may be interpreted as saying that the expected present value of net loss from an N-n stage process under an optimal policy is the minimum sum of expected loss associated with stage n and discounted expected loss from the remaining N-n-1 stages, provided an optimal policy is followed in the remaining N-n-1 stages for whatever state results from the decision in stage n.

To solve equation (2) by the dynamic programming technique we must first specify a terminal value for the process $f_{N+1}(i)$. This may be defined as zero since it is outside the planning period. Then,

$$V_i^k\ (N) = L_i^k, \qquad i = 1, 2, \ldots, M,$$

and

$$f_i\ (N) = \operatorname*{Min}_k L_i^k,$$

the minimum loss for state i in stage N, N years from the current period. In stage N-1,

$$V_i^k(N-1) = L_i^k + B \cdot \sum_{j=1}^{M} p_{ij}^k \cdot f_j(N),$$

and

$$f_i(N-1) = \operatorname*{Min}_k V_i^k(N-1) = \operatorname*{Min}_k[L_i^k + B \cdot \sum_{j=1}^{M} p_{ij}^k \cdot f_j(N)] \quad i = 1, 2, \ldots, M.$$

Note that for each level i of supply or input state, a value of k will be found that will minimize the right hand side of this equation. This may be written as $C_i(N-1)$, i = 1, 2, ..., M and represents the optimal subpolicy to follow in stage N-1 whatever the input state or beginning supply. To find the optimal overall strategy, it is necessary to proceed in this fashion, from the future toward the present, evaluating in succession $V_i^k(N)$, $V_k^k(N-1) \ldots V_i^k(1)$, i = 1, 2, ..., M, obtaining finally $f_i(1) = \underset{k}{\text{Min}}\ V_i^k(1)$, the minimum total discounted expected net loss for N stages for each of the M states. At each stage n, we will find the $C_i(n)$ associated with $f_i(n)$, finding finally the optimal overall policy $C_i(1)$. The sequence $C_i(1)$, $C_i(2)$, ..., $C_i(N)$, i = 1, 2, ..., M, is the optimal overall carryover strategy which minimizes the present value of expected loss over the N period planning horizon for whatever level of supply exists in each period.

This method of optimization is called dynamic programming. It consists of finding optimal subpolicies which include an ever increasing number of connected stages until the optimal policy is obtained (22, p. 85). Because the process if concerned with decision making under uncertainty, the optimal policy resulting from the multistage optimization is itself stochastic except for the first optimal decision (N time periods into the future). The remaining optimal decisions are determined as the stochastic process reveals itself (7, p. 156). This leaves us at the present looking N years into the future. We know the current value of the state variable S and the optimum decision k (a value of carryover C_t). In the next stage (stage 1), after the stochastic elements have been revealed, we again know an optimum choice, this coming from $f_i(1)$, etc.

It is now possible to restate the formal multistage problem and solution technique more compactly. Given a system with M possible states S_1, S_2, ..., S_M, the effect of a policy decision at time n is to transform state i into state j in a stochastic manner with probability p_{ij}^k. It is assumed that the probability that production is X_i is given by P_i and is the same for all time periods:

$$P_i(t) = \text{Prob}(X(t) = X_i(t)), \text{ and} \tag{4}$$

$$P_i(t) = P_i \text{ for all } t. \tag{5}$$

Then p_{ij}^k is the probability that the system will move from state i to state j in the next period when the alternative policy decision is k. Associated with each transition $S_i \longrightarrow S_j$ is a set of possible losses L_i^k, $k = 1, 2, \ldots, k$.

Then using the principle of optimality which states that for every n from 1 to N, an optimal substrategy from n to N must consist of optimal substrategies from N-n-1 to N, recursive equations typical of the dynamic programming technique may be developed to solve the decision problem. Define $f_i(n)$ as the discounted mathematical expectation of loss from stages n to N for input state i under an optimal policy:

$$f_i(n) = \underset{k}{\text{Min}} \left[L_i^k + B \cdot \sum_{j=1}^{M} p_{ij}^k \cdot f_j(n+1)\right] \quad \begin{array}{l} i = 1, 2, \ldots, M \\ k = 1, 2, \ldots, K. \end{array} \tag{6}$$

To find the optimal strategies it is necessary to evaluate the recursive equation iteratively from the future toward the present:

$$f_i(N) = \underset{k}{\text{Min}} \left[L_i^k + B \cdot \sum_{j=1}^{M} p_{ij}^k \cdot f_j(N+1)\right]$$

$$f_i(N-1) = \underset{k}{\text{Min}} \left[L_i^k + B \cdot \sum_{j=1}^{M} p_{ij}^k \cdot f_j(N)\right]$$

$$\ldots$$

$$f_i(2) = \underset{k}{\text{Min}} \left[L_i^k + B \cdot \sum_{j=1}^{M} P_{ij}^k \cdot f_j(3)\right]$$

$$f_1(1) = \underset{k}{\text{Min}}\ [L_i^k = B \cdot \sum_{j=1}^{M} P_{ij}^k \cdot f_j(2)]$$

$$i = 1, 2, \ldots, M$$
$$k = 1, 2, \ldots, K$$

with $f_j(N+1) = 0$, $j = 1, 2, \ldots, M$.

The optimal strategy for each of M possible initial supply levels in the current period is given by $C_i(1)$, $i = 1, 2, \ldots, M$, and is the carryover level which minimizes the sum of discounted expected losses for the N period planning horizon.

Convergence and an Infinite Planning Horizon

To this point in the discussion of the wheat carryover problem and the dynamic programming method of optimization, an economic planning horizon (the set of all stages representing intervals of time) of N periods has been assumed. It is probably more realistic to assume an infinite planning horizon because, as was mentioned earlier, to determine how much to store each year involves determining the division of total supply between the current period and all future periods. Fortunately, multistage decision problems of this nature converge to a constant optimal policy and a constant total loss function when the loss function does not vary with time and the total loss includes a discount factor of less than unity.

That is, for large values of n,

$$f_i(n) = f_i(n+1), \text{ and} \tag{7}$$

$$C_i(n) = C_i(n+1); \tag{8}$$

the optimal carryover k for the i^{th} supply level is the same whether N-n or N-n+1 periods remain in the planning horizon. According to Howard (23, pp. 84-85), convergence to an optimal policy usually occurs before convergence of $f_i(n)$.

Burt (10, pp. 40-43) has developed a procedure for determining if the policy resulting from a finite number of iterations has in fact converged to a fixed optimal policy. Following his presentation, the following quantities are defined so that equation (2) may be written in matrix form:

f(n), an NX1 column vector with i^{th} element $f_n(i)$;
C(n), an NX1 column vector with i^{th} element the value of k;
L(C), an NX1 column vector with i^{th} element L_i^k;
P(C), an NXN array of elements p_{ij}^k, the i^{th} element of C(n) determining k for the i^{th} row.

Now equation (2) becomes

$$f(n) = \text{Min } [L(C) + BP(C)f(n-1)], \; n = N, N-1, \ldots, 1. \tag{2'}$$

Then the solution of

$$(I - BP)\, X = L \tag{9}$$

for the vector X is the limit of the iterations of (2') under the assumption of a constant policy. The computational algorithm suggested by Burt is to:

1) carry out the iterations of (2) until convergence is indicated -- until $C_i(j)$ remains constant for several values of j;

2) solve the system of equations (9) for the constant policy X; and

3) obtain the solution to

$$Y = \underset{C}{\mathrm{Max}} \left[L(C) + B \cdot P(C)\ X \right] \tag{10}$$

If Y = X, the policy associated with X is the optimal policy for an infinite planning horizon. If Y differs from X, further iterations will result in another indicated convergence at which time another X is computed and compared to a new Y. This procedure is repeated until Y and X are equal, indicating the same constant policy.

The input data and assumptions necessary to cast the wheat inventory problem into a decision model of the type just presented and the results of the optimization procedure are given in the following chapter. The intent of this chapter has been to demonstrate the adaptability of dynamic programming to wheat inventory management as a multistage process and to outline the optimizing procedure and its rationale.

CHAPTER III

INPUT DATA AND THE DYNAMIC PROGRAMMING RESULTS

This chapter presents the results of the dynamic programming analysis of the wheat carryover problem. The empirical data necessary to cast the inventory problem into a serial multistage framework suitable for optimization using discrete dynamic programming is given first. This is followed by the results of the optimization procedure and the implications for policy decisions.

Input Data for Dynamic Programming Analysis

Primary input data for a discrete stochastic dynamic programming model of the type developed in Chapter II include: 1) a matrix of transition probabilities, and 2) a matrix of stage return values. Before the model can be subjected to the actual optimization procedure, consideration must also be given to: 1) the proper number of discrete states and alternatives to consider, and 2) a suitable interest rate to use to discount losses incurred in future time periods.

The purpose of using a discount factor is to provide an approximate means of comparing, on a comparable basis, the worth -- ideally, the utility -- of losses (social costs and storage costs) which are expected to occur in differing future time periods. Use of a discount factor provides a weighting procedure that assumes that losses expected to occur in time periods near to the present are of greater importance to the decision maker than an identical loss expected in more distant periods. A discount rate of six percent per period was used in this study. This is one evaluation of the time value, or "worth" in alternative uses, of public investment in a storage program.

Input States and Alternatives

As discussed in the preceding chapter, optimization of a decision process via the dynamic programming method involves serially combining suboptimal policies until an overall optimum is achieved. The "Principle of Optimality" ensures attaining an optimal policy for the problem as it is formulated and does so in a computationally efficient manner. In the discrete case, optimization at a given stage consists of finding the optimal policy for each of all possible states at that particular stage. That is, if in stage n there are K_1, K_2, ..., K_M decisions possible for each of M input states, the technique selects the alternatives, say k_1', k_2', ..., k_M'. Each k_i' is chosen from among K_i alternatives and represents the optimal decision to make at this stage if the input state is i.

For the wheat carryover model, each state (each value of i) represents a discrete level of supply which should be viewed as the midpoint of a class interval. Similarly, the amount of carryover represented by alternative k is the midpoint of a class interval. Then the number of discrete states, M, and the number of alternatives for each state, K_i, depend on the size of classes into which supply and carryover are divided as well as on the nature of the problem. For purposes of this analysis, the supply, production and carryover variables were each divided into 50 million bushel increments. State 1 represents a total supply of 1,100 million bushels (actually, the class 1,075-1,125 million bushels), state 2, 1,150 million bushels, etc. Alternative 1 (k = 1) represents a policy decision to have zero carryover into the next period, alternative 2, 50 million bushels, etc. The decision to use 50 million bushel increments resulted from several experiments which seemed to indicate that classes of this size provided sufficient approximation to a continuous function and gave convergence to a constant policy in relatively few iterations. Also, the computer algorithm used was easily adaptable to a problem of this size without causing undue increases in the time necessary to attain an optimum.

To clarify the discussion, several necessary assumptions and restrictions will be stated here and explained in later paragraphs.

1. Q^* = 1,550 million bushels. The quantity of wheat that would be taken off the market at the assumed equilibrium price in a "normal" year -- one in which the random effects of demand and supply variations are zero -- is taken to be 1,550 million bushels.
2. P^* = \$1.20 per bushel. The equilibrium price is assumed to be \$1.20 per bushel.
3. The range of values possible for aggregate yearly wheat production is from a low of 1,100 million bushels (the class 1,075-1,125 million bushels) to a maximum of 1,850 million bushels.
4. Maximum carryover for any period is assumed to be $S_t - Q^*$.

An assumption of Chapter II provided for carryover to be zero if total supply is less than the equilibrium quantity, Q^*. Assumption 4 above is a practical upper limit if supply is greater than Q^*.

Combining the two gives the following relationship for maximum carryover:

$$C_t^{Max} = \begin{cases} 0 & , \; S_t \le Q^* \\ \\ S_t - Q^*, & S_t > Q^*. \end{cases} \tag{1}$$

This restriction together with the decision to divide the supply and carryover variables into 50 million bushel classes suggested a model with 40 possible input states (representing supply values from 1,100 to 3,050 million bushels) and a maximum possible number of alternatives of 31 (reporting a carryover of up to 1,500 million bushels). The maximum values of 3,050 million bushels for supply and 1,500 million bushels for carryover seem adequate for any situation in which the United States wheat inventory is likely to find itself in the foreseeable future.

Table I shows the supply level corresponding to each state, the number of alternatives which must be considered for each state, K_i, and the maximum carryover level represented by the greatest K_i. Note that in each case, the difference between total supply and the maximum carryover is the equilibrium quantity, 1,550 million bushels.

The Transition Probabilities Matrix

Each element p_{ij}^k of the transition probabilities matrix P_{ij}^k gives the probability that the system will be transformed from state i in stage n to state j in stage n + 1. Because of the earlier assumption of stationarity, this holds for all possible values of n : P_{ij}^k is the same for all stages. Also stated earlier was the assumption that the

TABLE I

SUPPLY, MAXIMUM CARRYOVER AND NUMBER OF CARRYOVER ALTERNATIVES FOR EACH SUPPLY STATE

State (i)	Supply (S_t) (million bu.)	K_i	Max. Carryover (million bu.)	State (i)	Supply (S_t) (million bu.)	K_i	Max. Carryover (million bu.)
1	1,100	1	0	21	2,100	12	550
2	1,150	1	0	22	2,150	13	600
3	1,200	1	0	23	2,200	14	650
4	1,250	1	0	24	2,250	15	700
5	1,300	1	0	25	2,300	16	750
6	1,350	1	0	26	2,350	17	800
7	1,400	1	0	27	2,400	18	850
8	1,450	1	0	28	2,450	19	900
9	1,500	1	0	29	2,500	20	950
10	1,550	1	0	30	2,550	21	1,000
11	1,600	2	50	31	2,600	22	1,050
12	1,650	3	100	32	2,650	23	1,100
13	1,700	4	150	33	2,700	24	1,150
14	1,750	5	200	34	2,750	25	1,200
15	1,800	6	250	35	2,800	26	1,250
16	1,850	7	300	36	2,850	27	1,300
17	1,900	8	350	37	2,900	28	1,350
18	1,950	9	400	38	2,950	29	1,400
19	2,000	10	450	39	3,000	30	1,450
20	2,050	11	500	40	3,050	31	1,500

probability density function of yearly production, a random variable, is constant over time and is independent. That is, the probability that production will be some value X in time period t is the same for all values of t, and further, that this probability does not depend on any previously observed values of X.

Since $S_t = X_t + C_{t-1}$, the only stochastic element of supply is production. Then to find the probability of a particular supply level requires knowing the distribution of probability for yearly production. It was felt that since acreage is now largely a controlled, or at least a controllable variable, the variability in production that is of interest is that due to random yields -- the randomness resulting primarily from uncertain weather conditions.

To arrive at an estimate of the variability due to stochastic yields, the following procedure was used. For each of the years 1919-1967, the deviations about a linear trend in yields was multiplied by the acreage seeded in that year. This gave a measure of variability for each year. When adjusted to the assumed 1970 equilibrium level by adding 1,550 thousand bushels to each deviation and grouping the results into 50 million bushel classes, an empirical probability distribution was formed. This discrete distribution is given in Table II. The acreage and yield data upon which the calculations were based are given in Appendix A.

Again consider the relationship $S_t = X_t + C_{t-1}$. From this equation it is obvious that the only determinants of the supply level in period t are: 1) the known value of C_{t-1} (known at this point in time, but a function of total supply in period t-1), and 2) the random variable X_t. Then consider the probability that S_t will be some level,

TABLE II

EMPIRICAL PROBABILITY DISTRIBUTION OF PRODUCTION

Class	Lower Limit	Upper Limit	Midpoint	Frequency	Probability
	- million	bushels -			
1	1,075	1,125	1,100	2	.04167
2	1,125	1,175	1,150	1	.02083
3	1,175	1,225	1,200	2	.04167
4	1,225	1,275	1,250	3	.06250
5	1,275	1,325	1,300	4	.08333
6	1,375	1,375	1,350	1	.02083
7	1,375	1,425	1,400	3	.06250
8	1,425	1,475	1,450	2	.04167
9	1,475	1,525	1,500	7	.14583
10	1,525	1,575	1,550	5	.10417
11	1,575	1,625	1,600	7	.14583
12	1,625	1,675	1,650	4	.08333
13	1,675	1,725	1,700	3	.06250
14	1,725	1,775	1,750	3	.06250
15	1,775	1,825	1,800	0	.00000
16	1,825	1,875	1,850	1	.02083

say a, in period t.

$$\begin{aligned}\text{Prob}(S_t = a) &= \text{Prob}(X_t + C_{t-1} = a)\\ &= \text{Prob}(X_t = a - C_{t-1}).\end{aligned}$$

Since both a and C_{t-1} are known, the desired probability is easily determined.

For example, let a be 2,000 and C_{t-1} be 300. Then,

$$\begin{aligned}\text{Prob}(S_t = 2{,}000) &= \text{Prob}(X_t + 300 = 2{,}000)\\ &= \text{Prob}(X_t = 1{,}700).\end{aligned}$$

From this simple relationship, all values of P_{ij}^k can be determined. Notice that the value of k determines, for all i, the values of j. That is, the P_{ij}, j = 1, 2, ..., M, are the same for all values of i.

The Stage Return Values

The remaining data needed as input to the stochastic multistage decision model are the stage return values. For the wheat inventory model, the loss function given in equation (1) of Chapter II and repeated here is used to measure the stage return.

$$L(S,C) = \begin{cases} \displaystyle\int_{S_t}^{Q^*} (P_D - P_S)dq & , S_t \leq Q^* \\ \displaystyle R(C) + \int_{Q^*}^{S_t - C_t} (P_S - P_D)dq & , S_t > Q^*. \end{cases} \qquad (2)$$

To calculate net social cost at each stage transformation, it is necessary to assume market demand and supply schedules for wheat. The demand schedule used in this study is similar to the one developed by Tweeten (24, pp. 8-15) in 1965 with adjustments made to: 1) reflect changes that have occurred in the wheat economy during recent years, and 2) to force the curve through the assumed equilibrium point, $P^* = 120$, $Q^* = 1{,}550$. The linear function that was actually used is given in equation (3), with price measured in cents per bushel and quantity in milions of bushels.

The aggregate supply function was constructed as a two part function: the first segment has a constant price elasticity of supply of .3 and holds for quantities up to the equilibrium, the second segment is linear and also has a price elasticity of .3 at equilibrium. The market supply function is given in equation (4) as P_s.

$$P_D = 203.7 - .054Q, \text{ all } Q, \tag{3}$$

$$P_s = \begin{cases} \left(\dfrac{Q}{358.521}\right)^{\frac{10}{3}}, & Q \leq 1{,}550, \\ -280 + .258Q, & Q > 1{,}550. \end{cases} \tag{4}$$

The estimates that were assumed for the equilibrium price and quantity and the price elasticities of demand and supply (which together determine the demand and supply schedules) were not empirically or statistically determined, but represent a composite evaluation from several sources for the "proper" level for these parameters. The numeric values were established after examining the data and results from several research studies, both old and new (2, 24, 25, 26, 27), and after discussion with wheat marketing and policy researchers, extension economists, and economists and other officials of the grain trade industry.

The other term appearing in the loss function relationship is R(C), the cost of storage. For this analysis, the storage cost function is assumed to be linear with a marginal and average storage cost per bushel of 15 cents. Then,

$$R(C) = .15C, \tag{5}$$

where R is the total cost, in dollars, of storing C bushels of wheat for one period.

For the discrete problem, the stage return or loss function values are denoted by L_i^k and are the evaluations at discrete intervals of the continuous functions of (2) with the implicit terms replaced by (3) and (4).

Results of the Dynamic Programming Optimization

After the necessary input data were developed as explained above, the wheat inventory model was "optimized" using the dynamic programming approach. The computer algorithm employed optimized according to the "value iterative" method discussed by Howard (23, pp. 26-31). The results of the optimization are given in Tables III, IV, and V and Figure 2.

Table III gives the optimal carryover alternatives for the 40 states at each stage of the iterative process. After nine stages or

TABLE III

DYNAMIC PROGRAMMING OPTIMIZATION RESULTS:
OPTIMAL CARRYOVER ALTERNATIVES

	Stage								
State	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1
9	1	1	1	1	1	1	1	1	1
10	1	1	1	1	1	1	1	1	1
11	1	2	2	2	2	2	2	2	2
12	2	2	2	2	2	2	2	2	2
13	3	3	3	3	3	3	3	3	3
14	4	4	4	4	4	4	4	4	4
15	5	5	5	4	4	4	4	4	4
16	6	6	5	5	5	5	5	5	5
17	7	6	6	6	6	6	6	6	6
18	8	7	7	7	7	7	7	6	6
19	9	8	8	7	7	7	7	7	7
20	10	9	9	8	8	8	8	8	8
21	11	10	9	9	9	9	9	9	9
22	12	11	10	10	10	10	10	10	9
23	13	12	11	11	11	10	11	10	10
24	14	13	12	12	11	11	11	11	11
25	15	14	13	13	12	12	12	12	12
26	16	15	14	13	13	13	13	13	13
27	17	16	15	14	14	14	14	14	14
28	18	17	16	15	15	15	14	15	15
29	19	18	17	16	16	15	15	16	16
30	20	19	18	17	17	16	16	17	17
31	21	20	19	18	17	17	17	18	18
32	22	21	20	19	18	18	18	19	19
33	23	22	21	20	19	19	19	19	19
34	24	23	22	21	20	20	19	20	20
35	25	24	23	22	21	21	20	21	21
36	26	25	24	23	22	22	21	22	22
37	27	26	25	24	23	22	22	23	23
38	28	27	26	25	24	23	23	24	24
39	29	28	27	26	25	24	24	24	24
40	30	29	28	27	26	25	25	25	25

TABLE IV

DYNAMIC PROGRAMMING OPTIMIZATION RESULTS:
OPTIMAL CARRYOVER LEVELS

State	Supply	Carryover	State	Supply	Carryover
	- million bushels -			- million bushels -	
1	1,100	0.0	21	2,100	400
2	1,150	0.0	22	2,150	400
3	1,200	0.0	23	2,200	450
4	1,250	0.0	24	2,250	500
5	1,300	0.0	25	2,300	550
6	1,350	0.0	26	2,350	600
7	1,400	0.0	27	2,400	650
8	1,450	0.0	28	2,450	700
9	1,500	0.0	29	2,500	750
10	1,550	0.0	30	2,550	800
11	1,600	50	31	2,600	850
12	1,650	50	32	2,650	900
13	1,700	100	33	2,700	900
14	1,750	150	34	2,750	950
15	1,800	150	35	2,800	1,000
16	1,850	200	36	2,850	1,050
17	1,900	250	37	2,900	1,100
18	1,950	250	38	2,950	1,150
19	2,000	300	39	3,000	1,150
20	2,050	550	40	3,050	1,200

TABLE V

DYNAMIC PROGRAMMING OPTIMIZATION RESULTS:
MINIMUM LOSSES

State	Supply	Loss	State	Supply	Loss
	- million bu. -			- million bu. -	
1	1,100	568.4	21	2,100	534.3
2	1,150	521.3	22	2,150	562.4
3	1,200	479.2	23	2,200	592.8
4	1,250	442.3	24	2,250	625.1
5	1,300	411.2	25	2,300	659.7
6	1,350	386.2	26	2,350	695.8
7	1,400	367.8	27	2,400	732.0
8	1,450	356.4	28	2,450	770.0
9	1,500	352.5	29	2,500	810.1
10	1,550	354.3	30	2,550	851.9
11	1,600	358.3	31	2,600	895.2
12	1,650	364.2	32	2,650	939.3
13	1,700	374.0	33	2,700	984.1
14	1,750	386.1	34	2,750	1,030.6
15	1,800	399.7	35	2,800	1,078.6
16	1,850	416.9	36	2,850	1,128.0
17	1,900	436.9	37	2,900	1,178.8
18	1,950	457.3	38	2,950	1,230.9
19	2,000	480.4	39	3,000	1,283.0
20	2,050	506.2	40	3,050	1,332.4

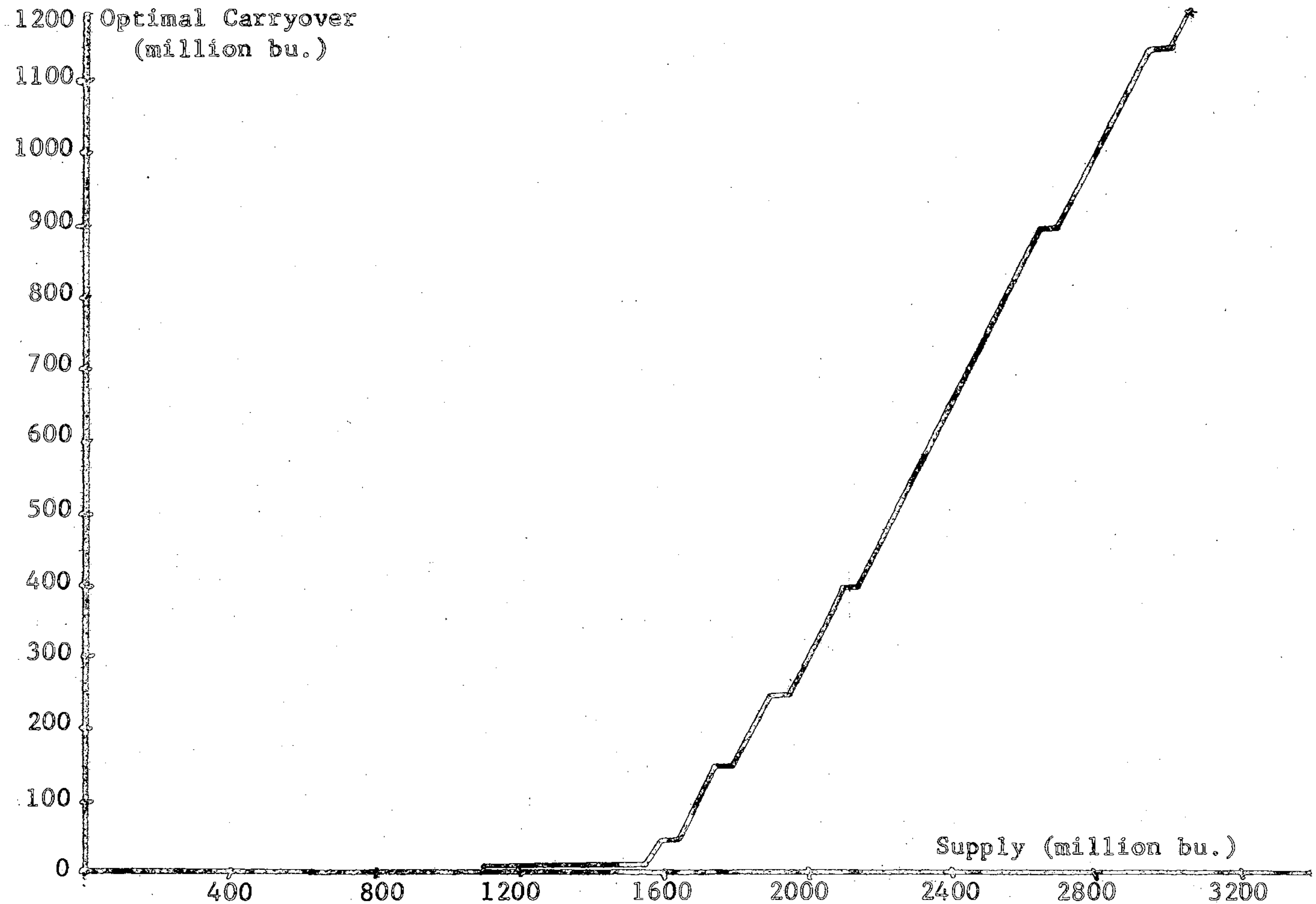

Figure 2. Continuous Representation of the Discrete Dynamic Programming Results Relating Optimal Carryover to Total Supply

iterations, the choice of alternatives at each stage converged to a constant optimal policy. The policy at stage 9 then, is the overall optimal policy for this model. The optimal carryover level for each of the 40 supply levels is shown in Table IV. Table V gives the total loss expected over nine stages if an optimal policy is followed at each decision period: each value is the total over nine stages of the expected value of the sum of net social loss discounted at a rate of six percent per period.

Figure 2, taken from Table IV, displays graphically the relationship between the possible levels of total supply and the associated optimal carryover. Portrayed in this manner, it is obvious that the results obtained from the discrete analysis can be adequately represented in geometric form by linear line segments. A reasonable mathematical approximation of this relationship between total supply and optimal carryover is given by:

$$C_t = \begin{cases} 0 & , S_t \leq 1550, \\ -1350 + .85S_t, & S_t > 1550. \end{cases} \tag{6}$$

As restricted by (1), there will be no carryover when total supply is less than the assumed equilibrium. For supply levels greater than the equilibrium quantity, 85 percent of the excess over 1,550 million bushels should be placed in storage in order to minimize the total expected losses over an infinite planning horizon.

Implications for Policy Decisions

As mentioned in the introductory chapter, one of the objectives of this study is to provide information of value to individuals and agencies responsible for managing aggregate wheat stocks. Table IV, in tabular form, and Figure 2 and equation (6) in geometric and mathematical form, purport to provide this type of information. Within the bounds placed on the model by various assumptions and the dynamic programming approach itself, these results tell how much wheat should be placed in storage each period in order to minimize total costs or losses. Specifically, the data say that if 85 percent of the excess of total available supplies over the 1,550 million bushel equilibrium is stored and the remaining 15 percent placed on the market, then total losses will be a minimum. It is important to note that these results show only what carryover should be after it is known how great supply is, supply being a random variable and known only after the random production value is known. The results do not show what target supply or production should be next period. This is one of the focal problems to be dealt with in the simulation analysis reported in the two subsequent chapters. The results reported above only show what carryover decision will minimize expected losses, after the decisions that influence production are made, and after the random effects of yields have made themselves known so that total supply is known.

Review of Assumptions and Restrictions

Because the objectives of this study include both methodological and empirical issues, it may be well to review the assumptions that have been made in order to make an aggregate wheat inventory model amendable to solution using the dynamic programming approach.

It seems possible to classify the various assumptions and restrictions into four somewhat interrelated groups: 1) those pertaining to assumed numerical parameter values, 2) those that relate to the measure of benefits used as the objective function or policy criterion, 3) the validity and reliability of certain statistical procedures, and 4) questions relating to the accuracy of describing wheat inventory management as a sequential decision model and to the accuracy of the discrete numerical analysis of the model.

With respect to the first group, values were assumed for an equilibrium price and an equilibrium quantity, for slope and intercept coefficients of demand and supply schedules, as well as their mathematical form, for the cost of storing wheat, and for the discount factor. In most cases, the specific numerical values used have only indirect statistical or theoretical reliability and were selected at the particular levels primarily because of a lack of better estimates.

For a discussion of the assumptions implicit in using the social cost concept as a policy criterion, reference is again made to the Tweeten and Tyner (19) publication mentioned earlier.

Issues of statistical importance center around the form of the empirical distribution of yearly production. The assumption of intertemporal independence may be somewhat inaccurate. Also, the procedure used to generate the empirical distribution is somewhat arbitrary -- other procedures could have been used resulting in a different distribution.

Finally, it is possible, from a national policy-making viewpoint, that wheat inventory management should not be viewed as a sequential decision process capable of being considered independently from other policy questions. Certain restrictions were also placed on the decision model to facilitate numerical analysis. Among these were the nondynamic assumptions of the stage returns and the transition probabilities, and the particular breakdown into discrete values.

These assumptions and restrictions do not necessarily make either the methodology or the results invalid, but additional research into these questions might prove enlightening. The simulation model presented in the subsequent chapter provides a sensitivity analysis of the decision rule given by the dynamic programming model.

Summary

Interest has centered in this chapter on the results of the dynamic programming optimization of the wheat inventory model developed in Chapter II. Data necessary to formulate the problem into a multistage framework include: 1) assumed demand and supply functions (which establish assumed normal equilibrium price and quantity values), 2) a matrix of transition probabilities developed from an empirical probability density function, and 3) stage return values developed from the loss function presented in implicit form in the preceding chapter and the assumed demand and supply functions.

The results of the optimization can be roughly translated into a conditional reserve management decision rule which says that (subject to the procedures used and within the limits imposed by the model and the data assumptions) net losses over an infinite planning horizon are minimized when approximately 85 percent of the excess of total supply is carried over into the next year.

The next three chapters attempt to generalize the conditional decision rule developed here by developing and operating a simulation model of appropriate portions of the aggregate wheat sector to study various reserve management policies.

CHAPTER IV

A SIMULATION MODEL TO STUDY RESERVE MANAGEMENT POLICIES

Introduction

The preceding chapter presented the results of a dynamic programming optimization of an aggregate wheat inventory model. The results from this type of analysis take the form of a conditional decision rule to govern carryover from one year to the next. This chapter attempts to generalize these results. The two primary objectives are to examine changes in relevant economic variables resulting from: 1) relaxing certain assumptions contained in the dynamic programming model, and 2) storage policies that do not follow the pattern suggested by the dynamic programming optimization.

With respect to the first objective, the key changes are to allow the demand variable to become stochastic and to incorporate acreage decisions into the model. As mentioned in Chapter I, one of the uncertainties associated with reserve stock management arises because export demand varies widely from year-to-year. However, the dynamic programming analysis assumed a static and deterministic aggregate demand schedule.

The decision model of the previous chapter also assumed that production varied only from the influence of the random variable yield: decisions concerning acreage were not considered. However, since acreage decisions are, at least to some extent, subject to influence by government planning, it seems desirable to further extend the results of the optimization by incorporating these decisions into the model.

Concerning objective two, the empirical goal is to examine the efficiency, compared to the dynamic programming results, of various other storage policies or rules of thumb similar to those that have been used or suggested in the past or that might be used or suggested in the future.

Characteristics of Simulation Analyses

The vehicle chosen to investigate these extensions or generalizations is a simulation model of the wheat economy. Each run of the simulation model generates a series of values for relevant economic variables whose magnitude and stability can be compared to the series generated by different runs having different conditions. Some of the variables considered significant include social cost, storage cost, farm income, wheat price and utilization and inventory levels.

Although this type of simulation analysis is a relatively new tool for economists, its efficacy in economic research has been well accepted. The still ascending popularity of simulation is generally attributed to characteristics that, when combined with the computational efficiency of computers, make the technique a very general tool, easily adaptable to a wide variety of problems. The results often provide more interpretable information than is readily available from other methods of analysis with the same amount of research effort.

The general approach of simulation as used in economic research is to construct a model which incorporates as many of the variables and relationships as is necessary to approximately characterize the conditions of a real economic system. A single run of the simulation model iteratively generates a stream of behavior for the endogenous variables that would be expected in the real world under similar conditions. Depending on the construction of the model, interest may center on the magnitude, or on the stability, or on the time paths and adjustment patterns of these variables. Changes in the behavior stream resulting from changes in original conditions or internal relationships for different runs can be observed and compared with any other run of the simulation model, thus providing information about the performance of the system in various situations. Results of a particular run are specific to the conditions set forth in that run, but many runs under many conditions allow the system to be studied in a general manner. Each run may be thought of as an experiment performed on a model, allowing investigations of hypotheses and conditionally determining the outcomes of various alternative courses of action without being forced to try the policies in the real system (15, p. 876).

The foregoing discussion of some of the essential features of simulation apply to both deterministic and stochastic models. An additional dimension is added to the analysis when Monte Carlo and simulation techniques are combined to make the model stochastic. If some variables or relationships can most properly (or expediently) be characterized as random, stochastically following some theoretical or empirical distribution of probability, then a value of each stochastic variable is randomly selected from the appropriate probability distribution during each iteration. A single run of the model usually consists of a great number of iterations -- the output or ending state of the system being used as the input or beginning state for the succeeding iteration. The behavior stream in this case is stochastic: the behavior of the relevant variables may be analyzed in terms of ordinary statistics such as means, measures of variation, and various order statistics, or in terms of frequency counts or histograms to indicate the likelihood of certain stochastic outcomes.

Characteristics of the Wheat Reserve Management Simulation Model

General characteristics of the simulation model developed to investigate questions relating to wheat carryover problems are presented in this section. Because the model varies somewhat for each type of carryover policy under consideration, the exact specification of the variables and parameters that are included will be discussed as the results and analyses for each run are presented. The framework of the model and the numerical values of the various parameters were established after due consideration to several factors including workability, *a priori* and statistical information and after experimentation to find those values and relationships that provided results that are reasonably consistent with what might be expected in the real system under the hypothesized conditions.

General Description of the Model

Several terms may be used to describe the model. For example, it may be appropriately described as an equilibrium model: price and utilization are determined by the economic requirement that the supply and demand quantity must be equal for each period. The model is also aggregative: total demand at each price is the horizontal summation of sector demands. The demand components considered include: 1) food, seed and industry (domestic), 2) feed, (3) export, and 4) stocks or carryover. The demand for stocks takes several forms throughout the analysis depending on the particular carryover policy being studied while the remaining demand elements are, for the most part, kept in the same form.

The model is also stochastic. Short-run (one-period) supply is the sum of carryover from last period and the current period's production, which is, in turn, the product of acreage and yield. Yield is a random variable, assumed to follow the discrete empirical distribution shown in Table VI. This distribution was developed by grouping the deviations about a linear trend into one-bushel-increment classes added to an assumed "normal" 1970 yield of 25 bushels per acre.

Export demand is also assumed to be influenced by random processes. The rationale for using a random variable instead of a strictly behavioral relationship to provide export demand estimates for each period is contained in two interrelated concepts. First, the record for reasonably simple predictive relationships developed from time series data is not good. The *a priori* estimates so obtained have been too often different from actual exports.

Secondly, predictive equations derived from time series assume that future export demand will be an outgrowth of, or at least closely related to, the past. However, there is little evidence to suggest that future world wheat trading patterns will be closely similar to past trading patterns. It is more likely that trading patterns will continue to show rather severe shifts, especially in the short run, particularly if common markets and market sharing arrangements continue to be the order of the day.

TABLE VI

EMPIRICAL PROBABILITY DISTRIBUTION OF YIELD

Yield Y	Probability g(Y)	Cumulative Probability G(Y)
- bushels per acre -		
21	.0625	.0625
22	.0208	.0833
23	.1250	.2083
24	.2083	.4167
25	.2292	.6458
26	.1250	.7708
27	.1042	.8750
28	.0833	.9583
29	.0208	.9791
30	.0209	1.0000

The actual simulation is accomplished by first formulating the model into a mathematical framework, then transforming these relationships into Fortran IV language to execute the computer simulation. Figure 3 is a simplified flow diagram of the model. The portion labeled A is executed each period (each period representing one marketing year), generating simulated time series for each variable. At the end of the run, summary statistics are computed for each of the key economic variables.

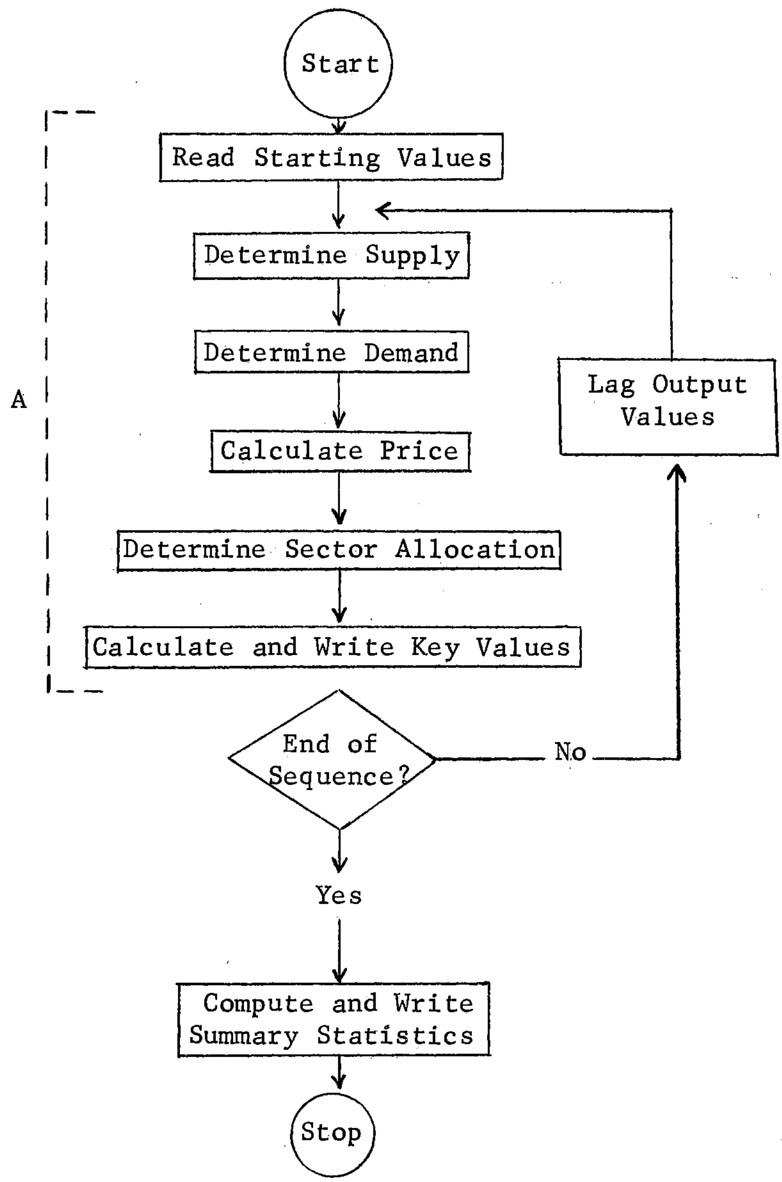

Figure 2. Continuous Representation of the Discrete Dynamic Programming Results Relating Optimal Carryover to Total Supply

Enough iterations are performed so that the series of key variables reflect a stable situation. There is no purely objective way to arrive at an optimal or even a satisfactory number of iterations, or a minimally acceptable series oscillation, either in absolute or percentage terms. To get some idea of the number of iterations required, several experiments were performed on the model, keeping cumulative means of several variables and plotting these against the number of iterations. Several of these are displayed in Appendix B and seem to show that the percentage improvement in stability of cumulative means becomes quite small after about 4000 iterations. Most of the actual runs used 4000 iterations except those for which the policies under consideration caused disequilibrium conditions and resulted in some type of excesses such as unacceptably large buildup of stocks.

To provide a starting point for the simulation analysis, the same supply-demand equilibrium conditions were assumed as were used in the dynamic programming analysis, namely an equilibrium supply quantity of 1550 million bushels is assumed to result from a normal acreage of 62 million and a normal yield of 25 bushels per acre. These values represent hypothetical "normal" values that would be expected to result under conditions of complete certainty and rational decisions as well as expected values (in the statistical sense) of the stochastic variables in the dynamic model.

Demand Relationships

The assumed demand relationships are given below.[1]

$$QH_t = 595. - .25P_t \qquad \text{all } P \tag{1}$$

$$QF_t = \begin{cases} 100. & P > 130 \\ 1270. - 9.0P_t & P \leq 130 \end{cases} \tag{2}$$

$$QE_t = 596.25 - 3.3125P_t + .75QE_{t-1} + \epsilon, \tag{3}$$

$$\epsilon = -200, -199, \ldots, -1, 0, 1, \ldots. 199.$$

QH is quantity consumed by the domestic food, seed, and industry sector,

QF is the quantity consumed by the domestic feed sector, and

QE is the quantity exported.

ϵ is the stochastic element of the export demand equation randomly chosen from the allowable set of numbers for each iteration. The effect of ϵ is to randomly shift the entire demand function horizontally right or left by an amount less than or equal to 200. Under equilibrium conditions ($P = 120$, $QE_t = QE_{t-1} = QE_{t-2} = \ldots$), $QE_t = 795$ and the

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[1]To avoid stating the units in which variables are expressed each time they are mentioned, these variables will be expressed in the following units throughout the remainder of this report -- in the text discussion and equations, and in tables and figures unless clearly noted otherwise:

a. quantities -- millions of bushels,
b. yield -- bushels per acre,
c. price -- cents per bushel,
d. acreage -- millions of acres,
e. incomes and costs -- millions of dollars.

effect of ϵ is to allow the quantity intercept to fall uniformly between 992.5 and 1392.5.

These sector demands give rise to the following aggregate demand, $Q_t = QH_t + QF_t + QE_t$:

$$Q_t = \begin{cases} 1291.25 - 3.5625P_t + .75QE_{t-1} + \epsilon & P > 130 \\ 2461.25 - 12.5625P_t + .75QE_{t-1} + \epsilon & P \leq 130. \end{cases} \qquad (4)$$

Figure 4 shows the sector and aggregate demand functions assuming $\epsilon = 0$ and an equilibrium value of 795 for QE_{t-1}, exports for the previous period. Table VII gives additional information about the characteristics of the demand schedules. This table shows, for three selected prices, the quantities that would be taken and the short-run price elasticities at each point for the individual and the aggregate demand functions.

Table VIII gives more detailed information about the export demand function characteristics. The table shows the effects on the quantity taken and on the demand elasticity of certain values for the random number ϵ and for QE_{t-1}, exports the preceding period. As ϵ and QE_{t-1} increase, resulting in a rightward shift of the linear demand function, exports become less elastic at each price.

The food and feed schedules, QH_t and QF_t, were adapted from those developed by Tweeten (4, pp. 8-15) in 1965, adjusted: 1) to reflect developments in usage patterns, and 2) to provide realistic and consistent results in the simulation process.

The export demand function was formulated to have several specific characteristics. It was felt that export demand can be most accurately

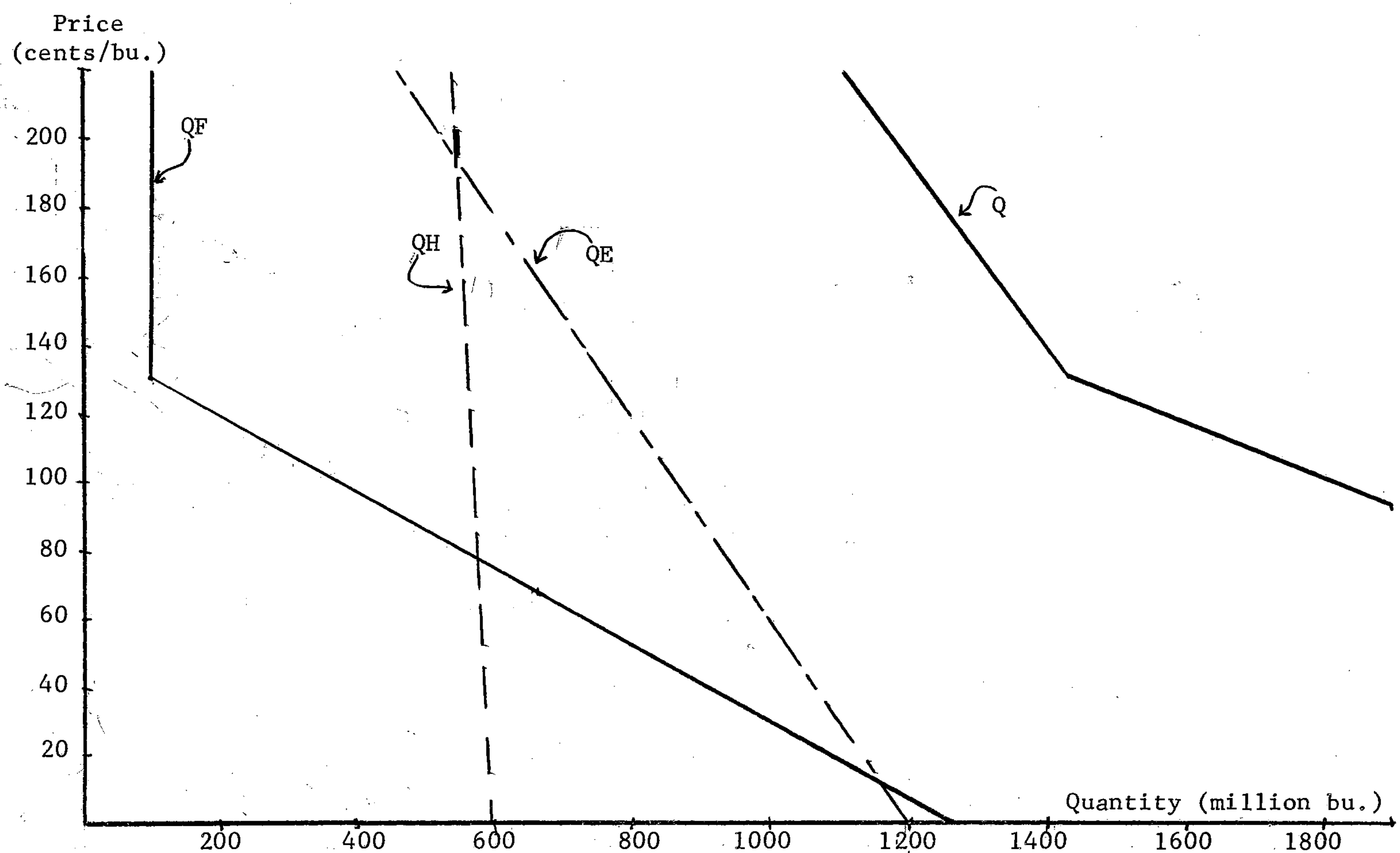

Figure 4. Food, Feed, Export and Aggregate Demands

and conveniently represented in the simulation process by a function that:

1) is linear,

2) is of the distributed lag form,

3) has prescribed short- and long-run price elasticities at the assumed normal or equilibrium prices and quantities, and

4) is stochastic.

TABLE VII

DEMAND FUNCTION CHARACTERISTICS: QUANTITY DEMANDED AND SHORT-RUN ELASTICITY AT THREE PRICES

	Price	Food	Feed	Export	Total
	100	570.0	370.0	861.3	1801.3
Quantity[a]	120	565.0	190.0	795.0	1550.0
	140	560.0	100.0	728.8	1388.8
	100	-.044	-2.432	-.385	-.697
Elasticity	120	-.053	-.5684	-.500	-.972
	140	-.063	0.0	-.636	-.359

[a]Units correspond to those given in footnote 1.

TABLE VIII

EXPORT DEMAND FUNCTION CHARACTERISTICS

Price	ϵ	QE_{t-1}[a]	QE_t[a]	Short-Run Elasticity	Long-Run Elasticity
100	-100	700	690	-.48	-1.92
		795	761	-.44	-1.74
		900	840	-.39	-1.58
	0	700	790	-.42	-1.68
		795	861	-.38	-1.54
		900	940	-.35	-1.41
	100	700	890	-.37	-1.49
		795	961	-.34	-1.38
		900	1040	-.32	-1.27
120	-100	700	624	-.64	-2.55
		795	695	-.57	-2.29
		900	774	-.51	-2.05
	0	700	724	-.55	-2.20
		795	795	-.44	-2.00
		900	974	-.45	-1.82
	100	700	824	-.48	-1.93
		795	895	-.44	-1.78
		900	974	-.41	-1.63
140	-100	700	558	-.83	-3.32
		795	629	-.74	-2.95
		900	708	-.66	-2.62
	0	700	658	-.70	-2.82
		795	729	-.64	-2.54
		900	808	-.57	-2.30
	100	700	758	-.61	-2.45
		795	828	-.56	-2.24
		900	908	-.51	-2.04

[a]Units correspond to those given in footnote 1.

The function was stochasticized by including the ϵ term in equation (3). For this function, ϵ was chosen to have a uniform distribution over the set (-200, -199, ..., 198, 199):

$$f(\epsilon) = 1/400, \ \epsilon = -200, -199, \ldots, -1, 0, 1, \ldots, 198, 199. \qquad (5)$$

The uniform distribution was chosen 1) because of a lack of prior information or reliable objective predictions about the nature of future export patterns, and 2) because, based on past data, this distribution of exports gave as much evidence of following the rectangular pattern as any other pattern.

The distributed lag formulation chosen was one in which the lag distribution has a single parameter. Using the general form assumed by Koyck to arrive at the linear equation:

$$Q_t = a + b_1 P_t + b_2 Q_{t-1}, \qquad (6)$$

then the short-run price elasticity is given by $b_1 \frac{\overline{P}}{\overline{Q}}$ and the long-run elasticity by $\frac{b_1}{1-b_2} \frac{\overline{P}}{\overline{Q}}$ where $\overline{P}$ and $\overline{Q}$ are the points on the function where the elasticities are to be measured. Equation (3) is of this form and has short-run elasticity of -.5 and long-run elasticity of -2.0 at the equilibrium price of 120 and an assumed export equilibrium of 795 million bushels (when $\epsilon = 0$, its expected value).

Supply Relationships

In the simulated system, the quantity of wheat available each period for all purposes, including demand by the three consuming sectors and for stock or carryover, is the sum of production in the current period and carryover from the previous period. Since current production is the product of yield and acreage, both supply and production are fixed amounts for the period: short-run supply is functionally represented as $X_t = a$, a vertical line independent of price for this period.

Yield in the production portion of supply is a random variable having the empirical density function given in Table VI. The procedure for developing this density function was discussed earlier. For a particular period, yield is selected by means of a random number generator which randomly assigns a value of Y according to the distribution of probability, g(Y).

Two different decision processes were used to determine acreage. One considered acreage to be determined as a functional market relationship. This corresponds to a free-market (on the supply side) situation, without government intervention. The other assumes the acreage decision to be a semiexogeneous policy decision, imposed on the decision unit (planters) from the outside, but not necessarily independent of market conditions. This corresponds to a situation in which acreage quotas are set by government policy makers based on inventory, anticipated needs, yields, and/or other policy considerations. For purposes of comparison, each carryover policy under consideration was simulated using both acreage decision conditions.

The functional relationship used is a linear, cobweb-type distributed lag equation which gives a short-run elasticity of .3 and a long-run elasticity of 1.0 at the equilibrium values of 62 million for acreage and 120 for price (equation 7).

$$A_t = .155\, P_{t-1} + .70 A_{t-1}. \tag{7}$$

To represent the decision process when acreage is set in accordance with predetermined policy goals requires knowing what these goals are. For purposes of this analysis, it is assumed that the proper goal is to set acreage at a level which is most likely to meet expected needs plus or minus an amount necessary to adjust stocks to a "desired" level. Three levels of carryover were rather arbitrarily selected to represent this "desired" carryover (200, 400 and 600 million bushels), and each reserve management policy is examined with acreages set to attempt to satisfy each desired carryover level. Notationally,

$$A_t = \frac{QP}{YP} + \frac{(C^* - C_{t-1})}{YP}, \tag{8}$$

where

C^* is desired or target carryover,

QP is the predicted or expected demand, and

YP is the predicted or expected yield.

The first term gives the acreage necessary to meet expected needs, the second is the adjustment in acreage required to achieve the desired stock adjustment. For purposes of simplicity, QP is the aggregate demand quantity at equilibrium conditions (QP = 1550), and YP is set at the expected value of Y, 25 bu./acre.

More exotic and complicated (or realistic, perhaps) decision rules could be devised to account for various exigencies in expected demand (as a critical world food situation, for example), or to incorporate more variables into the process, or to combine this policy-set acreage function with a market-determined function, etc. This latter might more accurately portray the effects of acreage allotments as currently used with their consequent slippages.

Miscellaneous Calculations

In addition to the demand-supply or price-quantity relationships, the values of several other variables were calculated each period to compare more completely the various reserve stock management policies. The most important of these are the variables associated with social cost or loss, with income derived from wheat production, and with costs of storage. This section discusses the procedures and parameters used to calculate those variables considered important.

As discussed in Chapter II, failure to utilize an equilibrium quantity of wheat at an equilibrium level may result in society incurring a loss composed of a social cost (or foregone benefit) plus a storage cost in the event available supply is greater than the equilibrium amount. As defined there, social cost can be measured by the area bounded above and below by the market demand and supply curves, and laterally by the equilibrium and utilized quantities.

The supply function used for the social cost calculation in the simulation process is the same market supply function used to calculate social cost for the dynamic programming analysis, and is given as equation (4) in Chapter III. This function has a constant elasticity of .3 through the point of normal equilibrium and is linear for quantities and prices greater than equilibrium. This market equation is the planning supply function and is assumed to remain constant for all periods.

The demand function which figures in the social cost calculation is the one which actually results from aggregating, for that period, the schedules of each consuming sector. Because of the dynamic and stochastic nature of the export demand equation, this aggregate function is different for each period and does not necessarily pass through the assumed equilibrium point. This formulation is necessary because the "normal" equilibrium (price = 120, quantity = 1550) is only that which would result if there were no uncertainties or dynamic elements. But the stochastic and dynamic elements of the export equation do shift demand and cause the equilibrium to vary from period-to-period.

When wheat is stored from one period to the next, it is assumed that the marginal and average cost of this storage is 15 cents per bushel, again the same value as used in the dynamic programming analysis given earlier. The sum of these costs is the total social loss incurred for the period. This loss function is given below as equation (9).

$$L(S,C) = \begin{cases} \int_{S_t}^{Q^*} (P_{D_t} - P_{S_t})dq & S_t \leq Q^* \\ R(C_t) + \int_{Q^*}^{S_t - C_t} (P_{S_t} - P_{D_t})dq & S_t > Q^* \end{cases} \tag{9}$$

where P_{D_t} is the inverse function of Q_t, given as equation (4) earlier:

$$Q_t = \begin{cases} 1291.25 - 3.5625\ P_t + .75\ QE_{t-1} + \epsilon & P_t > 130 \\ 2461.25 - 12.5625\ P_t + .75\ QE_{t-1} + \epsilon & P_t \leq 130, \end{cases} \tag{10}$$

and

$$P_{S_t} = \begin{cases} \left(\frac{Q_t}{358.521}\right)^{10/3} & Q_t > 1550 \\ -280 + .2581\ Q_t & Q_t \leq 1550 \end{cases}, \tag{11}$$

and

$$R(C_t) = .15\ C_t. \tag{12}$$

The income items recorded in the simulation analysis are total gross and net incomes from wheat production. Gross income is calculated as the product of price and quantity marketed each period, while net income includes a fixed per acre charge for cost of production. This charge represents all variable costs of production and is assumed to be a constant \$20 per acre for all levels of yield and acreage. Notationally,

$$GI_t = P_t \cdot Q_t, \tag{13}$$

$$NI_t = GI_t - 20\ A_t. \tag{14}$$

With regards to storage costs, no provision is made within the model to distinguish between the portions of storage costs borne by government agencies and by private concerns. This is because the portion of the storage function performed by each market entity is influenced by at least three factors:

1) Prevailing speculation within the private sector about future prices,
2) The particular inventory or reserve management model under construction, and
3) The goals (and methods of implementation) of other government farm programs.

These factors are obviously interrelated, not separable and measurable within the information framework available to the model; therefore, there is no way to apportion costs between the public and private sectors for each period. Aggregate storage costs, then, are calculated for period t as given in equation (12).

The Three Basic Reserve Management Models

Model I

The first inventory model (Model I) approximates a free market situation in which the stocking function is performed by the private sector according to supply-demand conditions within the industry. The quantity stored each period is determined by a functional relationship representing demand for stocks as an element of total demand. In terms of a reserve management policy, the operation of the model represents a "hands-off" policy. The proper policy with respect to reserves is to assume that private dealers and speculators will keep adequate reserves to meet emergency needs as they pursue normal profit-taking operations.

According to the assumed relationship, equilibrium carryover is 400 million bushels, and the price elasticity of demand for stocks is -1.2375 at a price of 120 and is perfectly inelastic for prices above 200. This makes a quantity of 70 million bushels a lower limit for carryover from one period to the next. The explicit forms of the demand-for-stocks function and the new aggregate demand which results from this formulation are given as QS_t and Q'_t in equations (15) and (16).

$$QS_t = \begin{cases} 70 & P > 200 \\ 895. - 4.125\, P_t & P \leq 200 \end{cases} \tag{15}$$

$$Q'_t = \begin{cases} 1361.25 - 3.5625\, P_t + .75\, QE_{t-1} + \varepsilon & P > 200 \\ 2186.25 - 7.7875\, P_t + .75\, QE_{t-1} + \varepsilon & 130 \leq P \leq 200 \\ 3356.25 - 16.6875\, P_t + .75\, QE_{t-1} + \varepsilon & P \leq 130. \end{cases} \tag{16}$$

Figure 5 is a static, one-period example of the operation of Model I. QS is the inventory demand function, Q the aggregate demand not including QS, and Q' the sum of Q and QS. The aggregate demand functions are drawn assuming $\epsilon = 0$ and $QE_{t-1} = 795$, their expected or normal values. If production in the current period (X_t) is 1475 and carryover from last period (C_{t-1}) is 775, then the total supply (S_t) available for all uses, including stocks, is 2250. The price will be established at 103 and, of the total supply of 2250, 470 (C_t) will be carried over into the next period and 1780 (q_t) allocated to the three consuming sectors.

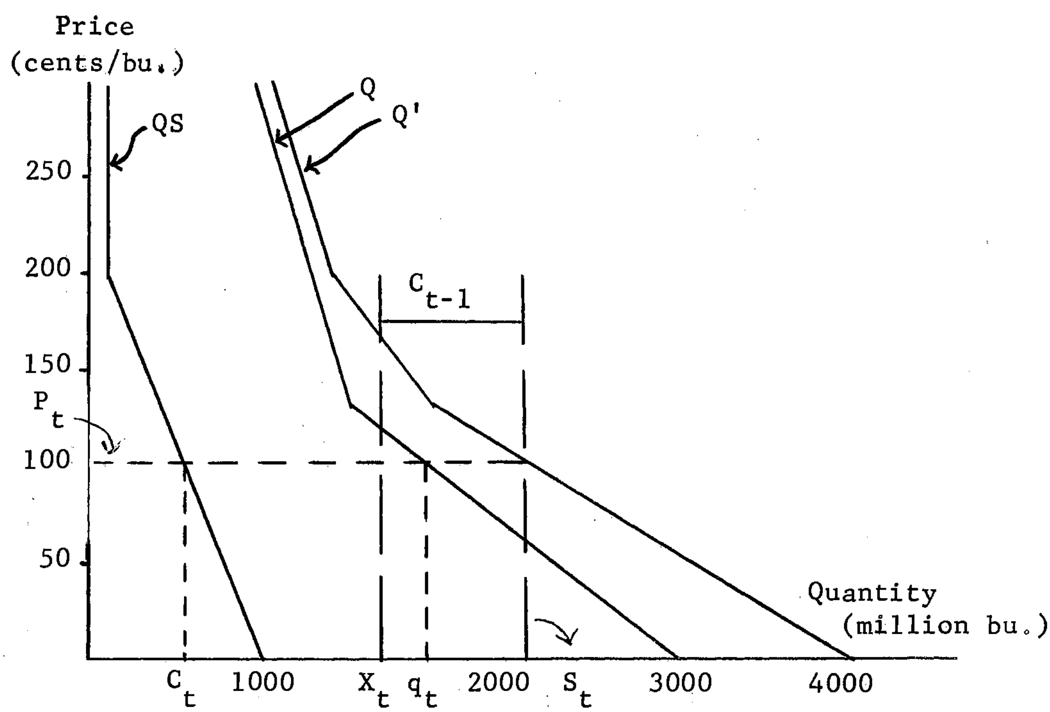

Figure 5. Static Example of the Operation of Model I

Model II

The second inventory model uses reserves as an instrument for the interrelated goals of domestic market stability and reserve supplies to meet emergency needs, both foreign and domestic. The operation of this model approaches that sought by the ever-normal granary idea of the 1930's as well as more current ideas as suggested by several recently proposed items of legislation. Typical of these are Senate and House of Representative bills S.2617, S.2743, S.2233 and H.R.14329 introduced during the First Session of the 90th Congress, but not enacted into law.

Each of these provides for adding to current stocks if reserves fall below an established safety level (about 20 percent of estimated export and domestic needs) provided the purchases can be made at or below a certain price (typically, 115 percent of the price support loan rate). Provisions are made to dispose of the stocks if the market price reaches a certain upper level (145 percent of the loan rate, or 100 percent of parity) even if carryover is expected to be below the established safety level. These provisions are designed to insulate stock adjustments from ordinary market operations during periods of fairly normal demand and supply conditions. The adjustments in stocks would be made only during years in which a shortage or surplus would otherwise result.

Model II provides for an adjustment to be made in inventory only if price reaches certain prescribed levels. Stocks will be decreased (and quantity marketed increased) when the prices reaches P^U, a predetermined distance above the equilibrium price, and will be increased (decreasing the quantity placed on the market) when price falls to P^L, a predetermined distance lower than the equilibrium price. Otherwise, the quantity marketed, Q, will be the amount produced, X_t.

With reference to Figure 6, if X_t is greater than Q_t^U, an adjustment in inventories will be made in an attempt to bolster price up to the lower limit P^L. For example, assume X_t is x_4. Then $x_4 - Q_t^U$ will be added to stocks (and subtracted from X_t) so that Q_t^U will be placed on the market at a price of P^L. Total carryover into the next period is $C_t = C_{t-1} + (X_t - Q_t^U)$. This operation is restricted by the condition that the model provides for an upper limit to stocks based on the previously discussed assumption that institutional factors will probably keep U. S. wheat inventory from being above 1 billion bushels. If the adjustment in stock necessary to increase price to P^L is enough to cause carryover to be above 1 billion bushels, the excess will be marketed, causing the market price to be below P^L.

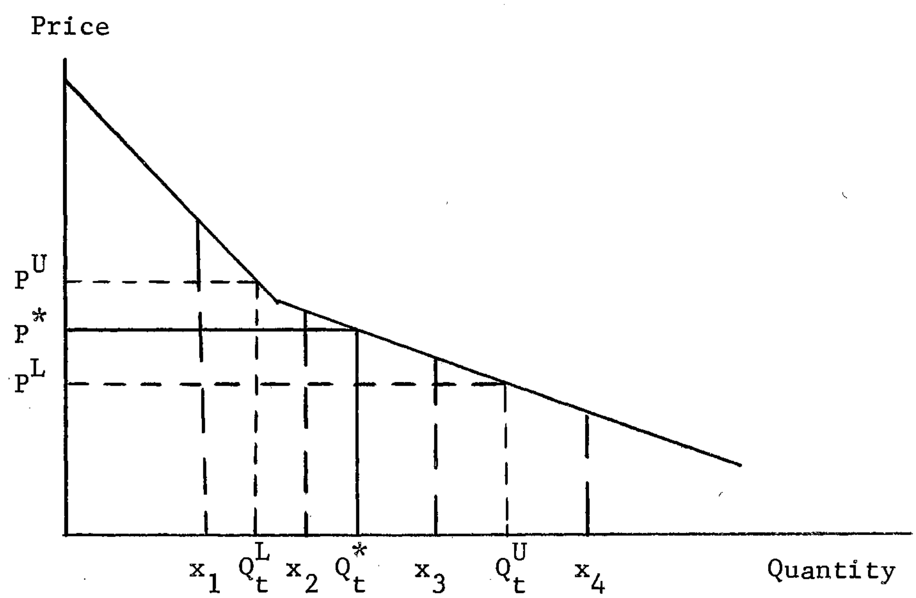

Figure 6. Illustration of the Operation of Model II

Note that P^U and P^L are fixed at prescribed levels, but because of the stochastic and dynamic nature of the demand function, Q_t^U and Q_t^L, being dependent on random and lagged values, are not the same for each period.

When production is less than Q_t^L, the opposite adjustment takes place. If X_t is x_1, $(Q_t^L - X_t)$ will be taken from reserves and placed on the market along with S_t so that Q_t^L will be sold at a price of P^U. Carryover into the next period will be $C_t = C_{t-1} - (Q_t^L - X_t)$ or $C_t = C_{t-1} + (X_t - Q_t^L)$. This operation is subject to the obvious restriction that stocks cannot be reduced below zero. If the stock adjustment $(Q_t^L - X_t)$ is greater than carryover from the last period, the quantity marketed is assumed to be $Q_t = X_t + C_{t-1}$ which will bring a price greater than P^U, and carryover into the next period will be zero.

When current production is between Q_t^L and Q_t^U ($X_t = x_2$ or x_3), the market operates without intervention, price is between P^U and P^L, and inventories remain at the same level.

Notationally, the inventory policy operates according to the following rules.

1. If $(X_t < Q_t^L$ and $(Q_t^L - X_t) < C_{t-1})$,

$$Q_t = X_t + (Q_t^L - X_t) = Q_t^L$$

$$P_t = P^U$$

$$C_t = C_{t-1} + (X_t - Q_t^L).$$

2. If $(X_t < Q_t^L$ and $(Q_t^L - X_t) \geq C_{t-1})$,

$Q_t = X_t + C_{t-1}$, $\qquad Q_t < Q_t^L$

$P_t \geq P^U$

$C_t = 0$.

3. If $(Q_t^L \leq X_t \leq Q_t^U)$,

$Q_t = X_t$

$P^U \leq P_t \leq P^L$

$C_t = C_{t-1}$.

4. If $(X_t > Q^U$ and $C_{t-1} + (X_t - Q_t^U) < 1{,}000)$.

$Q_t = X_t - (X_t - Q_t^U) = Q_t^U$

$P_t = P^L$

$C_t = C_{t-1} + (X_t - Q_t^U)$.

5. If $(X_t > Q_t^U$ and $C_{t-1} + (X_t - Q_t^U) \geq 1{,}000)$,

$Q_t = X_t + (1{,}000 - C_{t-1})$, $\qquad (Q_t \geq Q_t)$

$P_t \leq P^L$

$C_t = 1{,}000$.

The values chosen for P^U and P^L are arbitrary. For the actual simulation, several values were used, some of which provided for a uniform range around the assumed equilibrium price of 120, and some which provided for the purchasing price P^L to be closer to the equilibrium price than is the selling price P^U. This latter situation is one which might result if political pressure caused enforcement of policies which would call for supplies to be taken off the market when price dropped only slightly below a level considered desirable, but which prevented stocks from being sold except when price threatened to be exceedingly high. This would be a situation desirable to farmers, but could have undesirable results as discussed in a later section when the simulation results are presented.

Model III

The third model is designed to approximate and test the inventory rule suggested by the dynamic programming results as presented in Chapter III. As discussed there, to minimize total discounted expected losses over an infinite planning horizon, a reasonable approximation to the discrete results is provided by storing 85 percent of the amount by which total supply exceeds the normal demand quantity of 1550 million bushels. When the total supply quantity is less than 1550 million bushels, carryover will be zero.

To test this rule, Model III is programmed to operate as follows:

$$S_t = X_t + C_{t-1}$$

$$C_t = \begin{cases} \theta(S_t - Q^*), & S_t > Q^* \\ 0, & S_t \leq Q^* \end{cases} \qquad (Q^* = 1550)$$

$$Q_t = S_t - C_t,$$

where θ is the percentage of excess supply $(S_t - Q^*)$ which is to be carried over into the next period. Model III was run using several values for θ, ranging from 1.0 to .70.

Figure 7 gives a static, one-period example of the operation of Model III. Q is the aggregate demand function for the three consuming sectors when $\varepsilon = 0$ and $QE_{t-1} = 795$, their expected values. If production in the current period (X_t) is 1475 and carryover from last period (C_{t-1}) is 775, then total supply (S_t) available for all uses, including stocks, is 2250 and the excess of total supply over normal equilibrium (Q^*) is 700. If θ is .85 so that 85 percent of this excess is stored, carryover will be 595 and 1655 will be marketed at a price of approximately 112. Thus:

$$S_t = X_t + C_{t-1} = 1475 + 775 = 2250,$$

$$C_t = \theta(S_t - Q^*) = .85(2250 - 1550) = 595,$$

$$Q_t = S_t - C_t = 2250 - 595 = 1655.$$

Summary

This chapter has presented general characteristics and operational methods of three basic simulation models designed to examine U. S. wheat reserve management policies. The three models represent three quite different approaches to reserve management and somewhat different methods would be required to implement each into actual practice. The final chapter briefly discusses these issues along with pointing out certain advantages and disadvantages of each model.

The next two chapters present the results of the simulation performed on each model. The chapter immediately following gives the results for the simulation of the three basic reserve management policies and some variations of each policy, while the results of certain changes made within the framework of the model are given in Chapter VI.

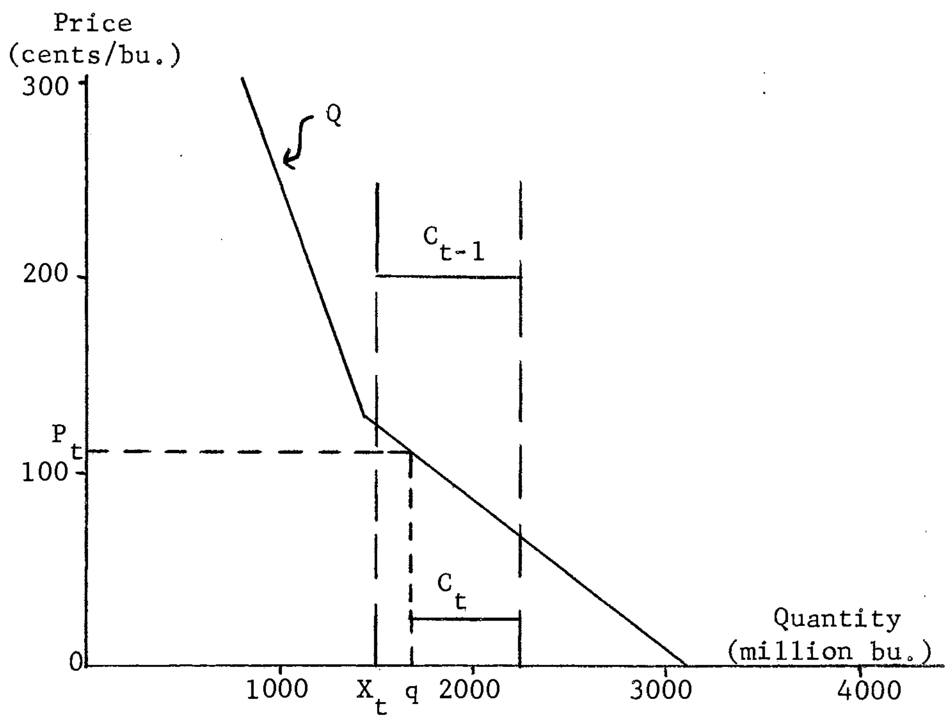

Figure 7. Static Example of the Operation of Model III

CHAPTER V

RESULTS FROM SIMULATING THE THREE BASIC MODELS

Introduction

The task of this chapter is to present the results of the simulation analysis. It was mentioned in Chapter I that one of the empirical objectives of this study is to provide information that will make possible "better" reserve management decisions. The policy maker can be expected to base his choice of reserve management policies, not just on net social cost, but on several measures such as the mean and variance of prices, receipts, net income, production and other variables. Accordingly, a number of such variables are presented in this chapter from the simulated operation of the wheat economy under the three stock management models described in the previous chapter.

The primary purpose of the simulation model is to examine and compare various reserve management policies; therefore, the most important end products of this section are the comparisons, under conditions as similar as possible, of the three basic inventory models. The fact that each model was run under different assumptions about supply determination and that several variations of the reserve management policies were simulated in two of the three basic models makes exhaustive presentation of the results quite cumbersome.

To make as many of these comparisons as meaningful as possible, a certain schema is at least loosely followed in the subsequent pages.

First the results considered significant from each basic model are presented. The results from one model are compared internally (where applicable) to show the differences resulting from each variation of that inventory rule. Comparisons are then shown among the results obtained via each supply determination assumption. Following this, the reserve management policies are compared with one another on as similar bases as possible.

A number of variations of the working model were also examined, and the results are shown in the next chapter. These variations served to check on the flexibility of the model and its ability to be adapted to changing conditions in the real system and to examine its sensitivity to assigned parameter values. These changes include a different supply function, different export demand elasticities, and different overall equilibrium levels resulting from a lower assumed value for the equilibrium level for exports.

Model I Results

The operation of inventory Model I, where stocks are determined according to a functional relationship representing private market interests, was simulated under the four different supply situations. The first situation considers supply to be "market determined" according to equation (7), Chapter IV, while the other three situations correspond to supply set exogenously as in equation (8), Chapter IV, with C^*, the desired or target carryover set at 200, 400, and 600 million bushels. In the following discussion of the results, and in the accompanying tables, these four situations are designated as situations I_M, I_2, I_4, and I_6, respectively. Table IX shows five summary measures

TABLE IX

SELECTED MODEL I SIMULATION RESULTS: MEANS, COEFFICIENTS OF VARIATION, MINIMUMS AND MAXIMUMS OF TWELVE VARIABLES

Variable[a]	Model I_M	Model I_2	Model I_4	Model I_6	Variable[a]	Model I_M	Model I_2	Model I_4	Model I_6
Acreage					Stocks				
Mean	62.4	55.6	62.1	68.9	Mean	396.6	360.6	396.4	428.3
Std. Dev.	3.3	2.5	2.3	2.1	Std. Dev.	57.6	62.5	58.6	52.8
Coef. of Var.	5.2	4.4	3.7	3.1	Coef. of Var.	14.5	17.3	14.8	12.3
Minimum	52.8	49.3	56.7	62.9	Minimum	126.6	115.9	150.1	196.6
Maximum	76.2	65.4	72.0	78.1	Maximum	556.1	516.9	532.8	576.9
Yield					Price				
Mean	25.0	25.0	25.0	25.0	Mean	120.8	129.6	120.9	113.0
Std. Dev.	2.0	2.0	2.0	2.0	Std. Dev.	14.1	15.4	14.4	13.0
Coef. of Var.	8.0	8.1	8.1	7.9	Coef. of Var.	11.7	11.9	11.9	11.5
Minimum	21.0	21.0	21.0	21.0	Minimum	81.8	91.4	87.5	76.7
Maximum	30.0	30.0	30.0	30.0	Maximum	186.9	189.6	181.2	169.8
Production					Net Income				
Mean	1,560	1,390	1,552	1,720	Mean	626.7	683.1	625.1	558.4
Std. Dev.	147	132	139	146	Std. Dev.	169.1	182.9	179.8	171.9
Coef. of Var.	9.4	9.5	9.0	8.5	Coef. of Var.	26.9	26.8	28.8	30.8
Minimum	1,108	1,098	1,197	1,321	Minimum	174.7	281.6	234.8	105.3
Maximum	2,278	1,836	2,013	2,135	Maximum	1,288.9	1,315.0	1,232.3	1,177.6
Food					Gross Income				
Mean	564.8	562.5	564.7	566.7	Mean	1,875.3	1,794.4	1,867.8	1,935.7
Std. Dev.	4.0	4.6	4.2	3.8	Std. Dev.	195.7	211.3	207.4	195.3
Coef. of Var.	.70	.81	.75	.68	Coef. of Var.	10.4	11.8	11.1	10.1
Minimum	548.3	547.6	549.7	552.5	Minimum	1,354.6	1,268.0	1,382.6	1,440.3
Maximum	574.6	572.2	573.1	575.8	Maximum	2,674.9	2,512.3	2,513.2	2,681.7
Feed					Social Cost				
Mean	203.8	154.9	204.4	258.2	Mean	17.3	41.9	15.0	55.1
Std. Dev.	88.8	67.2	89.9	103.7	Std. Dev.	26.6	38.5	23.8	59.5
Coef. of Var.	43.6	43.4	44.0	40.1	Coef. of Var.	153.7	92.0	158.4	108.0
Minimum	100.0	100.0	100.0	100.0	Minimum	0.0	0.0	0.0	0.0
Maximum	534.1	447.7	482.8	580.0	Maximum	276.5	425.0	186.7	299.6
Export					Total Loss				
Mean	792.0	672.9	783.5	895.8	Mean	76.8	96.0	74.5	119.4
Std. Dev.	119.3	108.7	118.2	122.3	Std. Dev.	28.3	34.6	24.8	64.6
Coef. of Var.	15.1	16.2	15.1	13.6	Coef. of Var.	36.8	36.0	33.4	54.1
Minimum	433.3	374.0	471.0	560.1	Minimum	34.9	43.7	35.2	34.5
Maximum	1,166.5	1,025.3	1,104.4	1,227.0	Maximum	308.4	442.4	219.6	373.0

[a]Units correspond to those given in footnote 1 of Chapter IV.

on each of the twelve variables from 4000 simulated periods for the four situations.

Comparison of situations I_2, I_4, and I_6 demonstrates the effects of assuming that the demand for all components, including stocks, follows one functional form, dependent upon price, while supply is determined according to other considerations. Having supply and demand components determined according to different considerations does not cause a true disequilibrium condition to exist in the sense of price and quantity failing to tend toward stable average values, but the supply determination for the I_2 and I_6 cases do cause new equilibriums to be established. To see why this is so, consider the expected value for supply each period:

$$E(S_t) = E(X_t + C_{t-1}) = E(A_t \cdot Y_t + C_{t-1})$$

$$= \frac{QP + C^* - C_{t-1}}{YP} \cdot E(Y_t) + C_{t-1},$$

and since $E(Y_t) = 25 = YP$,

$$E(S_t) = QP + C^* - C_{t-1} + C_{t-1}$$

$$= 1550 + C^*, \qquad (QP = 1550).$$

Then the expected supply when the desired carryover, C^*, is 200, 400, and 600 million is:

$$E(S_t^{200}) = 1750,$$

$$E(S_t^{400}) = 1950,$$

$$E(S_t^{600}) = 2150.$$

Only $E(S_t^{400})$ is consistent with the normal equilibrium price of 120 and total quantity of 1950 (1550 for the three consuming sectors plus 400 for stocks). In this case the desired carryover is also the expected value for stocks under the normal equilibrium values. The lower average quantity supplied must result in a higher average price and the greater quantity in a lower price. It is interesting to note that in the actual simulation, total supply averaged 1751, 1949, and 2149 for the three situations.

It is possible to use the elasticities of demand to roughly predict the price and sector allocations resulting from these quantity changes. For example, from Tables VII and VIII (Chapter IV), it is possible to approximate an aggregate long-run price flexibility which is about -.60 for values near equilibrium. This means that the new equilibrium price should be established about six percent lower in the case of quantity supplied being 10 percent higher, and six percent higher as the quantity supplied drops 10 percent to 1750. Average price actually increased 7.2 percent for situation I_2 and decreased 6.5 percent for situation I_6. Also, the change in exports of approximately 14 percent is consistent with the price changes of 7.2 and 6.5 percent and the long-run export demand elasticity of about -2.0. Similar comparisons are possible for the other demand sectors.

The situations represented by I_2 and I_6 are disequilibrium in the sense that acreage is always set as if to reach a carryover level that will not, on the average, be achieved. From the stock relationship $QS_t = 895. - 4.125P_t$ (this is appropriate because price never reached 200, the upper limit for this equation to apply), it is seen that the 200 million bushel average would be reached only with an average price of 168 while the 600 million bushels figure requires an average price of 72.

These situations might roughly correspond to the events that could occur under a free selling market with acreage quotas set using incorrect demand information or estimates. For example, acreage might be set at a level consistent with believing that the market will demand 200 million bushels for inventory at an established "fair price" of \$1.20 per bushel, but the industry wishes to stock 400 million bushels at this price, thus driving the market price higher. The consuming sectors use less at this higher price, and stocks are less than would be taken at the \$1.20 price but greater than the 200 million bushels expected to be taken.

The income figures resulting from these situations are somewhat interesting. The lower quantity-higher price equilibrium of situation I_2 causes net income to increase by 9.3 percent, as compared with situation I_4, while gross income decreases by 3.9 percent. The results are similar for the higher quantity-lower price equilibrium of situation I_6 where net income is 10.7 percent less and gross income is 3.6 percent greater, again as compared with situation I_4. Using the coefficient of variation as a measure of stability, the higher income values are associated with greater stability for both measures of income, but the opposite is true for net income when the standard deviation is used as the indicator of stability. As average gross income increases from 1794 to 1868 to 1936 (for I_2, I_4, and I_6, respectively), the coefficient of variation decreases from 12 to 11 to 10 and the standard deviation also decreases from 211 to 207 to 195. As average net income increases from 558 to 625 to 683 (for I_6, I_4, and I_2), the coefficient of variation decreases from 31 to 29 to 27 but the standard deviation increases from 172 to 180 to 183.

Situations I_2 and I_6 both result in an average social cost figure several times that of I_4, probably indicating that the "struggle" to reach an impossible equilibrium causes the quantity marketed to vary markedly from the equilibrium quantity.[1] When storage cost is added to calculate the total loss, approximately the same average absolute differences remain, but the relative differences average much less.

It is possible to compare the results of situations I_M and I_4 to show the responses resulting from the two acreage determination rules. Both situations theoretically give the same expected price and quantity equilibriums so that the differences that show up must be caused either by random influences or by the different supply determination methods. Because the average price and quantity figures are very similar for the two situations, any substantial differences must be due mostly to the way in which acreage is set.

Table X, adapted from Table IX, compares these two situations. The figures in the differences columns show the measure from I_4 subtracted from that of I_M : $I_M - I_4$. The percentage figures give the measure from situation I_M as a percent of that from I_4 : I_M/I_4. Thus,

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[1]In the calculation of social cost for this model, the market demand function is the aggregate of the three consuming sectors, and the market supply function is that established earlier as the fixed "planning" supply function. The equilibrium quantity is established by the intersection of these curves, and the quantity utilized is calculated as $Q_t + X_t + C_{t-1} - C_t$ so is the quantity taken by the three consuming sectors. Thus the equilibrium quantity corresponds to that which must be established because of the acreage determination rule so that the figures given correctly calculate social cost as defined earlier.

if the measure (mean, standard deviation, coefficient of variation, or range) on a particular variable is greater for situation I_M than for I_4, the difference shows as positive and the percentage as greater than 100.

TABLE X

SELECTED MODEL I SIMULATION RESULTS: COMPARISON OF SITUATIONS I_M AND I_4

	Mean		Std. Dev.		Coef. of Var.		Range
	Diff.	Pct.	Diff.	Pct.	Diff.	Pct.	Diff.
Acreage	0.3	100.4	1.0	143.5	1.5	140.5	8.1
Production	8.0	100.5	8.0	105.7	0.4	104.4	354.0
Stocks	0.2	100.1	- 1.0	98.2	-0.3	98.0	46.8
Price	-0.1	99.9	- 0.3	97.9	-0.2	98.3	11.4
Net Income	1.6	100.2	-10.7	94.0	-1.9	93.4	116.7
Gross Income	7.5	104.5	-11.7	94.4	-0.7	93.7	189.7
Social Cost	2.3	115.3	2.8	111.8	-4.7	97.0	23.1
Total Loss	2.3	103.1	3.5	114.1	3.4	110.2	89.1

[a]Differences are $I - I_{400}$, percentages are I/I_{400}: if the measure from I is greater than from I_{400}, the difference shows as positive and the percentage is greater than 100.

[b]Units correspond to those given in footnote 1 of Chapter IV.

Although there is no test to evaluate the statistical significance of any differences, most seem to be quite small. In the means column, only average social cost shows a sizable difference, at least in percentage terms. With regards to measures of variability, market determination of supply (situation I_M) results in more variable acreage but more stable income measures.

Model II Results

Model II, which provides for stocks to be used to maintain price within certain prescribed limits,[2] was simulated using the same four supply determination conditions as Model I. These situations are designated as II_M, II_2, II_4, and II_6 in the following discussion.

As discussed earlier, the values for P^U and P^L, which establish the price range within which the market operates without reserve stock adjustments, were arbitrarily chosen. Table XI shows the 12 different price range situations which were simulated. From this table it is seen that for situation 1 the range is actually zero so that inventory adjustments are used to force the equilibrium price to prevail whenever possible. Also, only situations 1, 6, 10, 11, and 12 are "equilibrium" situations in the sense that there is a uniform range around the assumed normal equilibrium price of 120. In the other seven situations, the "selling price," P^U, is further removed from the equilibrium price than is P^L, the "buying price." For situations 2, 3, 4, and 5, P^L, the buying price is also the assumed equilibrium price of 120, but the

[2]Or conversely, uses market price to indicate a potentially undesirable situation (shortage or surplus) that warrants prevention via an inventory adjustment.

TABLE XI

TWELVE MODEL II PRICE RANGE SITUATIONS

Situation	P^U - P^L	Spread	Spread From P^* P^U	Spread From P^* P^L
		- cents per bushel -		
1[a]	120 - 120	0	0	0
2	125 - 120	5	5	0
3	130 - 120	10	10	0
4	140 - 120	20	20	0
5	150 - 120	30	30	0
6[a]	125 - 120	10	5	5
7	130 - 115	15	10	5
8	140 - 115	25	20	5
9	150 - 115	35	30	5
10[a]	130 - 110	20	10	10
11[a]	140 - 100	40	20	20
12[a]	150 - 90	60	30	30

[a]For situations 1, 6, 10, 11 and 12, P^U is the same amount above as P^* as P^L is below P^*. These are referred to as the equilibrium situation of Model II. For the remaining situations, P^U is farther above P^* than P^L is below P^*. These are referred to as the disequilibrium situations of Model II.

selling price, P^U, varies from 125 to 150. Thus P^*, the equilibrium or normal or desired price, also acts as a floor below which price will not fall except when stocks reach an upper limit of 1000 as explained earlier.

The fact that Model II was simulated using twelve price ranges each under four supply determination conditions means that there are 48 different situations to consider and compare for Model II alone. This obviously makes reporting of all measures on all variables impossible: this section attempts to present those results from the simulation runs which are economically significant and which will show how this reserve stock management policy performs under various conditions. The first results presented consider only the "equilibrium" situations, 1, 6, 10, 11, and 12, simulated for 4000 periods, and show comparisons between assuming acreage is market determined as opposed to assuming acreage is "policy-set."

Situations Having a Uniform Range Around P^*

Tables XII and XIII show the means and coefficients of variation for eight series under the four supply determination conditions. Table XIV shows the percentage occurrence of certain events indicative of the performance of the model. Column 6 of Table XIV shows the percentage of periods for which stock adjustments were not called for because the current period's production fell within the range established by P^U and P^L, the upper and lower price limits. This percentage is a sort of "degree of insulation from the market" provided reserve stocks by the particular inventory rule. Columns 3 and 4 show the percentage of time stock adjustments were used successfully to keep price within the

TABLE XII

SELECTED MODEL II SIMULATION RESULTS, EQUILIBRIUM SITUATIONS: MEANS OF EIGHT VARIABLES

Situation[a] (P^U - P^L)	Model	Means[b]							
		Acreage	Production	Price	Stocks	Net Income	Gross Income	Social Cost	Total Loss
	II_M	62.3	1558	120.5	559	629	1875	21.3	105.2
1	II_2	60.0	1502	121.6	249	626	1827	24.2	61.6
(120-120)	II_4	61.0	1526	120.2	425	614	1834	26.3	90.0
	II_6	61.3	1534	119.8	617	612	1838	25.9	118.5
	II_M	62.1	1553	120.3	541	620	1863	16.5	97.7
6	II_2	60.2	1508	121.3	243	619	1824	24.1	60.7
(125-115)	II_4	61.0	1526	120.2	426	609	1828	25.1	88.9
	II_6	61.1	1529	120.0	622	608	1830	24.6	117.9
	II_M	62.1	1551	120.2	567	617	1859	15.8	100.8
10	II_2	60.3	1509	121.2	243	614	1820	27.2	63.6
(130-110)	II_4	60.8	1521	120.4	430	606	1822	27.0	91.6
	II_6	60.8	1522	120.4	630	606	1822	27.0	121.5
	II_M	62.5	1561	120.9	232	624	1874	28.6	63.4
11	II_2	62.6	1568	119.3	183	602	1855	32.2	59.7
(140-100)	II_4	63.1	1582	118.5	371	594	1856	33.2	88.8
	II_6	63.2	1581	118.5	570	595	1859	33.7	119.2
	II_M	62.8	1571	121.6	79	633	1890	41.1	53.0
12	II_2	65.1	1629	116.7	122	581	1884	45.3	63.7
(150-190)	II_4	65.7	1646	115.7	305	572	1887	49.9	95.7
	II_6	65.9	1649	115.6	502	570	1889	52.0	127.3

[a]Situation numbers correspond to those given in Table XI.

[b]Units correspond to those given in footnote 1 of Chapter IV.

TABLE XIII

SELECTED MODEL II SIMULATION RESULTS, EQUILIBRIUM SITUATIONS: COEFFICIENTS OF VARIATION OF EIGHT VARIABLES

Situation[a]		Coefficients of Variation							
($P^U - P^L$)	Model	Acreage	Production	Price	Stocks	Net Income	Gross Income	Social Cost	Total Loss
	II_M	5.0	9.8	10.8	61.2	34.5	13.0	199.6	57.4
1	II_2	12.2	14.6	5.9	73.9	27.7	15.4	137.3	82.1
(120-120)	II_4	13.6	15.7	1.2	48.9	27.0	15.7	119.1	54.1
	II_6	13.5	15.6	1.1	33.6	27.3	15.7	115.3	36.2
	II_M	4.6	9.4	9.6	65.6	28.8	11.0	247.0	66.2
6	II_2	11.0	13.7	6.8	67.9	23.7	12.9	138.5	76.5
(125-115)	II_4	12.1	14.5	3.8	43.4	21.8	12.8	128.3	51.5
	II_6	11.9	14.4	3.8	29.5	21.9	12.7	126.7	36.5
	II_M	4.2	9.2	9.4	59.7	26.2	10.0	192.7	57.1
10	II_2	9.8	12.8	8.5	60.8	22.9	11.3	137.2	72.8
(130-110)	II_4	10.5	13.3	6.8	37.4	21.0	11.0	136.6	51.0
	II_6	10.5	13.3	6.8	25.5	21.0	11.0	136.6	38.3
	II_M	5.5	9.8	14.5	96.8	34.1	12.1	193.0	96.1
11	II_2	7.8	11.3	12.3	66.6	28.4	10.9	143.9	79.0
(140-100)	II_4	8.3	11.7	11.2	35.8	26.4	10.2	143.9	55.6
	II_6	8.4	11.7	11.3	23.6	26.4	10.3	145.3	42.0
	II_M	6.0	10.0	17.8	140.4	41.0	13.8	157.6	123.5
12	II_2	5.6	9.9	14.9	75.0	35.4	11.5	131.1	90.6
(150-190)	II_4	6.3	10.3	14.0	34.5	33.9	10.9	131.7	66.1
	II_6	6.6	10.5	14.1	21.9	34.0	10.9	134.9	52.3

[a]Situation numbers correspond to those given in Table XI.

TABLE XIV

SELECTED MODEL II SIMULATION RESULTS, EQUILIBRIUM SITUATIONS: PERCENT OCCURRENCE OF SIX PRICE-RELATED EVENTS

Situation[a] (P^U - P^L)	Model	Percentage Occurrences — Column Number[b] 1	2	3	4	5	6
	II_M	8.6	8.6	75.7	75.7	15.7	0
1	II_2	12.6	12.6	87.4	87.4	0	0
(120-120)	II_4	2.4	2.4	97.6	97.6	0	0
	II_6	0	0	96.9	96.9	3.1	0
	II_M	9.3	7.3	31.7	27.7	10.0	23.3
6	II_2	9.4	6.3	32.4	34.0	0	27.3
(125-115)	II_4	1.5	1.4	36.9	35.7	0	26.0
	II_6	0	0	38.0	34.6	1.5	26.0
	II_M	5.4	3.1	23.0	19.8	6.1	48.0
10	II_2	6.0	3.2	24.3	23.8	0	48.7
(130-110)	II_4	0.3	0.2	26.9	24.4	0	48.5
	II_6	0	0	27.0	23.9	0.6	48.5
	II_M	19.1	5.7	12.9	11.0	0.1	70.3
11	II_2	7.4	2.5	13.8	11.3	0	72.4
(140-100)	II_4	0.1	0.1	16.1	12.1	0	72.7
	II_6	0	0	16.2	12.1	0	71.7
	II_M	37.1	8.2	5.3	4.7	0	81.8
12	II_2	7.9	2.1	6.2	5.4	0	86.3
(150-190)	II_4	0	0	7.6	6.1	0	86.3
	II_6	0	0	7.6	6.1	0	86.3

[a]Situation numbers correspond to those given in Table XI.

[b]

Column Number	Percentage of:	Column Number	Percentage of:
1	Zero Inventory	4	Price equal to P^L
2	Price greater than P^U	5	Price less than P^L
3	Price equal to P^U	6	Price between P^U and P^L.

established limits while columns 2 and 5 indicate how often price could not be maintained within this range because of the limits of zero and 1000 placed on the size of the reserves.

Looking first at the results of situation II where acreage is market determined, the means in Table XII show several responses as the range within which the market operates without reserve stock adjustments increases. Some of these are apparently due to the increased range itself, others to interactions of the increased range and certain limits imposed within the model.

For example, as the spread between P^U and P^L increases from 20 to 60, both price and production increase slightly. This comes from the interaction of the price spread with the kinked feed demand schedule. When the price spread is 20 or less, price reaches above 130 only if stocks reach the maximum allowable value of 1000. But when the spread is above 20, price reaches the 130 level more often, causing the vertical segment of the feed demand curve to become more important. Because the quantity of feed demanded is always at least 100 no matter how high the price, the average value for total demand is greater than it would otherwise be. This in turn causes production to increase slightly because of the assumed positive-dependent relationship of acreage to price. As both price and production increase, gross income must increase, and in this case, the higher production costs of slightly greater acreage are not sufficient to keep net income from also increasing.

Average social cost shows a similar response which is quite consistent with what might be expected. As the price spread increases slightly from zero, reserves are used with some success to keep the quantity marketed fairly close to the equilibrium level, but as the spread becomes greater, the quantity actually used is free to vary further from the assumed equilibrium causing average social cost to increase. The fact that average total social loss decreases is directly connected to the average size of stocks held as the spread increases. The values shown for average stocks decrease as the range between P^U and P^L widens. This is largely a chance happening. When stocks fell to zero (or a very low level), it was possible for this value to hold for many periods because the price spread was so wide that only seldom did production vary enough to cause an adjustment in reserves to be called for. This can be seen from Table XIV which shows, for example, that when P^U and P^L were 150 and 90 respectively, 28.1 percent of the time (37.1 - 8.2), inventories were zero but production was not sufficient to warrant adding anything to reserves according to the inventory policy being followed. When many of these low or zero values were encountered, the average value for stocks was naturally low. Since total loss includes a storage charge, total loss decreased as average stocks became very low.

From Table XIII, it is seen that when the coefficient of variation is used to measure variability, greater stability of all series is achieved when the upper and lower price limits are 130 and 100 respectively. This is an indication that, according to the model, a price spread either way of 10 from the desired or equilibrium price provides for sufficient flexibility and size of stocks to maintain a reasonably stable marketing and production situation. This price range also resulted in less chance of zero inventory when acreage is market determined.

When supply is determined by setting acreage at a level designed specifically to result in production sufficient to cover the predicted consumption needs of the current period plus a desired carryover (situations II_2, II_4, and II_6), several different responses are noted. In situation II_M where acreage is determined via the distributed lag equation (equation (7), Chapter IV), not only is price dependent upon the level of production through the negative price coefficient of the demand function, but production is also directly dependent upon price through the positive relationship of acreage to price. In situations II_2, II_4, and II_6, acreage is not directly dependent upon price but on the deviation of actual carryover from desired carryover (equation (8), Chapter IV).

Table XIII shows generally more variability in acreage and production and less variability in the price and income series for the three "controlled-supply" situations than for the market-supply situation. This is an indication that tying acreage to the deviation of actual from desired carryover makes possible larger and more immediate adjustments in production. The result is a more orderly market in terms of price and income.

From Table XV it is possible to deduce why increasing the price spread results in lower price and inventory and in more production as seen in Table XII. Table XV shows the quantity ranges established by P^L and P^U above and below Q_t^*, the quantity established by the actual demand function evaluated at P^*, the equilibrium price of 120. These are the ranges within which production can vary from Q_t^* without a reserve stock adjustment being necessary to keep price between P^U and P^L (Figure 6, Chapter IV).

TABLE XV

QUANTITY RANGES ESTABLISHED BY THE TWELVE PRICE RANGE SITUATIONS OF MODEL II

Situation[a]	Price Limits[c]		Quantity Ranges Around Q_t^{*c}	
	P^U	P^L	Below	Above
1[b]	120	120	0	0
2	125	120	62.8	0
3	130	120	125.6	0
4	140	120	161.2	0
5	150	120	196.9	0
6[b]	125	115	62.8	62.8
7	130	115	125.6	62.8
8	140	115	161.2	62.8
9	150	115	196.9	62.8
10[b]	130	110	125.6	125.6
11[b]	140	100	161.2	251.3
12[b]	150	90	196.9	376.9

[a]Situation numbers correspond to those given in Table XI.

[b]Situations 1, 6, 10, 11, and 12 are those referred to as equilibrium situations in the text.

[c]Units correspond to those given in footnote 1 of Chapter IV.

When the price range falls completely within the lower, less sloping portion of the demand curve, the quantity ranges are the same above and below Q_t^*. When the price spread widens so that P^U is above 130 and is in the more steeply sloped segment (which results from the zero elasticity portion of the feed demand function), the quantity range above is greater than below. This is necessarily the case because a given quantity change results in a greater price change when the demand function is more negatively sloped (the coefficient of price is smaller negatively). Then assuming an equal distribution of X_t above and below Q_t^*, X_t will fall below Q_t^L less often than above Q_t^U so that there will be more frequent withdrawals from than additions to reserve stocks.

The result is that when the price spread is as in situations 11 and 12, average stocks are lower than for situations 1, 6, and 10. Since acreage is tied to the deviation of C_{t-1} from C^*, average acreage must increase because this deviation is positive more often than it is negative. More acres means higher production which drives price down allowing quantity taken to increase.

The income series for situations II_2, II_4, and II_6 show that increasing the price spread results in generally higher gross but lower net income. The changes are not great, net income falling about seven percent from high to low and gross income rising about three percent.

Average social cost is substantially greater in all three cases for wider price spreads as the quantity used deviates further, on the average, from the equilibrium quantity for the period. When social cost and storage cost are added to arrive at the total loss, the values are nearly the same for all price spreads because the higher social costs associated with the wider ranges are just offset by lower storage costs from small reserve stocks.

Table XIII shows that for the three controlled supply situations a wider price spread results in greater stability for the acreage, production, stocks and gross income series, and less stable price, net income and total loss series.

As has been previously explained, the simulation model operates somewhat differently, depending on the supply determination conditions followed. Although this makes it somewhat difficult to present many valid comparisons of results between the two assumed methods of determining acreage, a few generalizations are possible.

For producers, both the level and stability of the income series, particularly net income, are probably most important. From Tables XII and XIII it appears that allowing acreages to be market determined results in higher but less stable income than having an outside force influence supply by setting acreage according to the specific rule employed. The market-determined acreage condition (situation II_M) also resulted in lower average social cost, an item unimportant to producers but important to society in general.

Another item of importance to society is the performance of the reserve management policy with respect to how effectively it operates to maintain reserves. From Table XIV it is seen that under situations II_M and II_2, there is substantial chance of zero inventory and price being above the arbitrarily established limit even when the spread between P^U and P^L is fairly wide. Setting the target carryover at 400 and 600 nearly does away with this problem according to the simulated results.

Before any conclusions can be drawn from these results it should be observed that this inventory policy is one of outside management; that is, it assumes an outside agency operating according to specific rules, and in essence this agency is sometimes called upon to manipulate demand within certain limits. Of the two methods of supply determination, one also assumes the intervention of an outside agency, one does not. So situation II_M represents a situation involving outside management on the demand side but not on the supply side. Situations II_2, II_4, and II_6 assume management on both the demand and supply sides.

From Table XIV it appears that management on the demand side only results in a fairly high chance that reserves will not be adequate according to the reserve management rules being followed. By allowing supply as well as demand to be managed, reserves will be adequate in nearly all cases as long as the target carryover is close to 400 million bushels. The managed supply situations also provide some degree of income stability. But the desirable features of adequate reserves and stable incomes are purchased, in a sense, with somewhat lower incomes, undesirable to producers, and higher social cost, undesirable to society in general.

These statements again point out that: 1) simulation is not an optimizing procedure because it cannot choose a best situation, and 2) an economic trade-off exists between different measures of different economically important variables.

Situations Having a Nonuniform Range Around P^*

The seven simulated situations of Model II where the spread between P^U and P^* is greater than between P^L and P^* present additional problems in data reporting. These problems stem from the fact that these are not true equilibrium situations; therefore, some series can show continually increasing or decreasing tendencies so that the summary statistics do not stabilize and as a result are not very useful. Because of these unstable tendencies, the seven situations discussed here were simulated for only 50 periods. Fifty iterations are not enough for the random effects of this type of Monte Carlo simulation to be averaged out and the random variables to approach their expected values so that the nonstability problems are confounded with the nonrandom problems for these situations. For these reasons, not much reliance can be placed on the actual values of the summary statistics.

In spite of the data reporting difficulties, some interesting and potentially significant trends or tendencies appear for which general observations are in order.

From Table XV and from Figure 6 of Chapter IV, it is possible to deduce the nature of the biases built into these seven situations of Model II. In all seven "nonequilibrium" cases, the range below the pseudoequilibrium quantity of Q^* is greater than the range above Q^*. This promotes two obvious biases. First, to the extent that reserve management is able to keep price between P^U and P^L (and quantity marketed between Q_t^L and Q_t^U), and even assuming a uniform distribution of X_t between Q_t^L and Q_t^U, price must average above P^*. This results in less quantity taken and lower demand -- operating through the lagged variable in the export demand function. Secondly, conversely to the situation explained earlier, there is more likelihood of additions to than withdrawals from stock because production does not have to fall as far below Q^* to cause an addition as it does to fall above Q^* to cause a withdrawal. Therefore, there is a definite bias toward large reserves: the rule is operating in a manner whereby it is relatively "easier" to take wheat off the market, increasing the market price and increasing stocks than it is to remove wheat from stock and place it on the market, decreasing the market price.

These biases are apparent in Table XVI which shows low quantity values, high prices and large stocks, with the severity of the bias increasing with the severity of the disequilibrium. One would expect this type of situation to be desirable for producers -- demand being manipulated mostly upward and higher average prices. This is demonstrated in a general way by the net income values of Table XVI which are generally higher than for situations 1, 6, 10, 11, and 12 discussed earlier.

The social cost values for these situations are quite interesting in that they compare quite favorably with those resulting from the equilibrium situations, and in fact, are quite often lower. The total loss values are somewhat higher because of the high storage costs that result from the large, upwardly biased reserve stock levels.

Probably the single most interesting result shown is the ability of "managed supply" to preserve, within limits, a rather orderly market in spite of the biases discussed. Figure 8 shows the time paths for 35 periods taken by stocks under the four acreage determination conditions for one price spread situation. As is apparent from this figure, when acreage is determined by the market conditions, stocks soon reach the 1000 maximum and remain at that level throughout most of the iterations. This means that reserves are not able to provide much stability to the market. However, by having supply "managed" also, stocks

TABLE XVI

SELECTED MODEL II SIMULATION RESULTS, NONEQUILIBRIUM SITUATIONS: MEANS OF EIGHT VARIABLES

Situation[b] (P^U - P^L)	Model	Means[a]							
		Acreage	Production	Price	Stocks	Net Income	Gross Income	Social Cost	Total Loss
	II_M	63.4	1575	122.5	698	651	1919	31.1	135.8
2	II_2	60.4	1507	122.2	242	631	1839	13.8	50.1
(125-120)	II_4	60.5	1508	122.1	440	630	1840	13.8	79.7
	II_6	60.5	1508	122.1	640	630	1840	13.6	109.6
	II_M	61.0	1531	118.0	934	577	1797	26.5	166.6
3	II_2	58.4	1456	124.4	291	640	1809	22.8	66.5
(130-120)	II_4	58.4	1456	124.4	491	640	1809	22.8	96.5
	II_6	58.5	1458	124.3	689	640	1810	21.9	125.4
4	II_M	63.5	1585	122.8	943	655	1936	22.4	163.9
(140-120)	II_2	56.3	1403	128.1	344	665	1791	36.7	88.4
	II_4	56.3	1403	128.1	544	665	1791	36.7	118.4
	II_6	56.4	1406	128.0	741	665	1794	34.8	145.9
	II_M	63.0	1598	121.6	965	664	1927	28.0	172.7
5	II_2	54.0	1353	131.5	394	684	1771	55.2	114.3
(150-120)	II_4	54.3	1353	131.5	593	684	1771	55.2	144.3
	II_6	54.5	1359	131.2	788	684	1775	51.6	169.9
	II_M	62.2	1561	120.5	808	626	1870	15.7	136.9
7	II_2	60.5	1509	127.1	238	627	1837	15.9	51.7
(130-115)	II_4	60.5	1509	122.1	438	627	1837	15.9	81.7
	II_6	60.5	1509	122.1	638	627	1837	15.9	111.7
	II_M	62.3	1568	120.4	855	629	1876	17.9	146.1
8	II_2	58.8	1465	124.9	282	647	1823	21.5	63.8
(140-115)	II_4	58.8	1465	124.9	482	647	1823	21.5	93.8
	II_6	58.8	1464	124.9	682	647	1823	21.5	123.8
	II_M	63.0	1577	122.0	962	652	1918	21.1	165.4
9	II_2	57.0	1421	127.8	325	666	1807	32.1	80.9
(150-115)	II_4	57.0	1421	127.8	525	666	1807	32.1	110.9
	II_6	57.0	1421	127.8	725	666	1807	32.1	140.9

[a]Units correspond to those given in footnote 1 of Chapter IV.

[b]Situation numbers correspond to those given in Table XI.

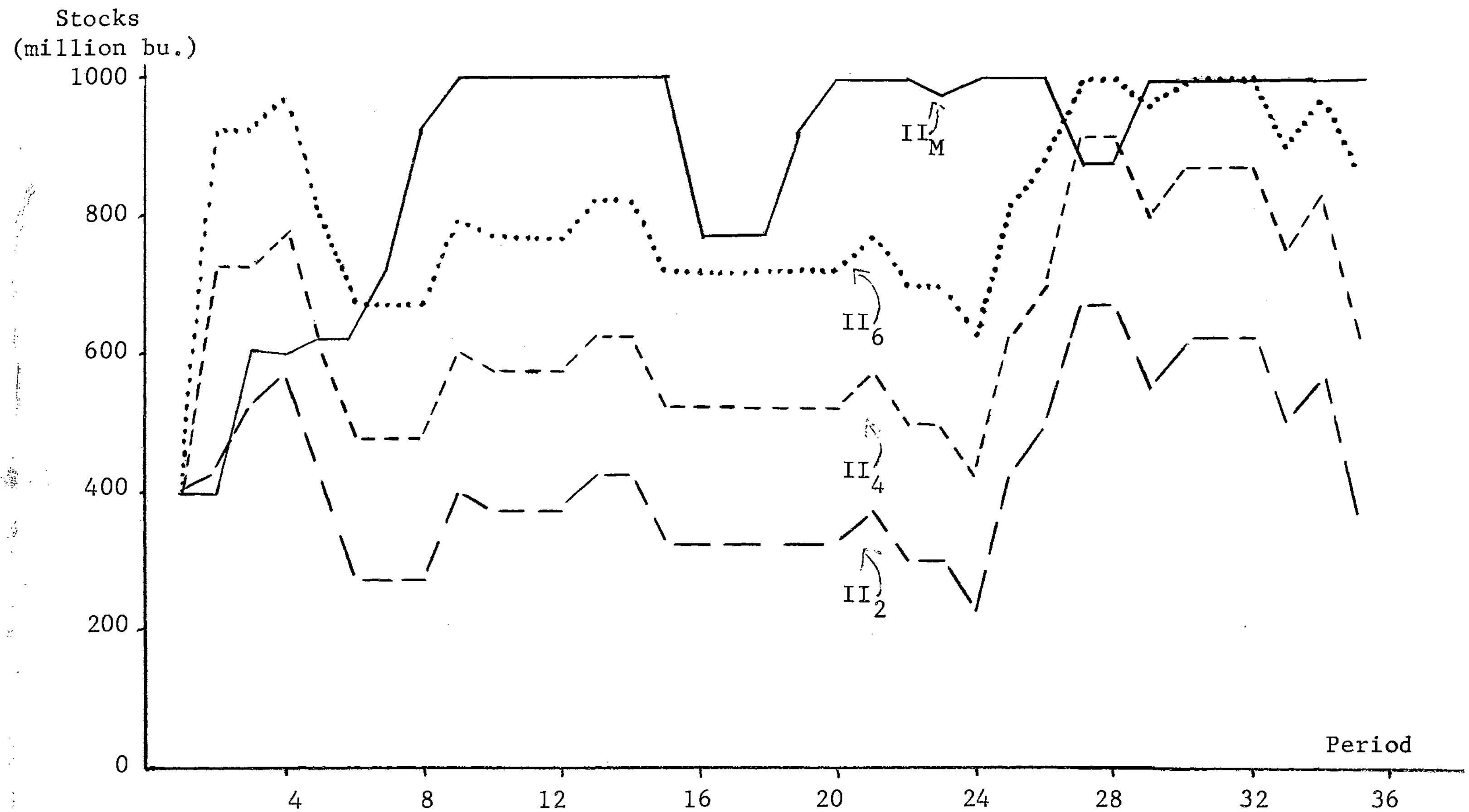

Figure 8. Carryover for 35 Periods, Model II

are mostly kept below the upper limit in spite of the bias discussed earlier. The biases are still apparent: stocks never reach as low as the desired level and there are more additions to than withdrawals from reserves. But this acreage determination method does manage to keep reserves viable enough to provide some degree of market stability. Table XVII, which shows the coefficients of variation associated with the variables whose means are displayed in Table XVI, also demonstrates these tendencies. In particular, the coefficients of variation for price and income are substantially lower for situations II_2, II_4, and II_6 than for II_M.

Model III Results

Model III which was designed to approximate and test the results of the dynamic programming analysis of Chapter III, and which establishes as carryover from one period to the next a portion of the amount by which total supply exceeds the assumed equilibrium demand of 1550 million bushels, was simulated using the same four supply determination conditions as Models I and II. These four situations are designated as III_M, III_2, III_4, and III_6 in the following discussion.

Five values were arbitrarily chosen for θ, the fraction which determines the portion of "excess supply" treated as carryover. The term excess supply refers here to the amount by which production plus carryover from the previous period exceeds 1550. If excess supply is found to be negative, it is treated as zero so that carryover into the next period is zero and the quantity marketed is the quantity produced in the current period. The five values chosen for θ are: 1.0, .90, .80, .75, and .70. Note that when $\theta = 1.0$, the quantity marketed will

TABLE XVII

SELECTED MODEL II SIMULATION RESULTS, NONEQUILIBRIUM SITUATIONS: COEFFICIENTS OF VARIATION OF EIGHT VARIABLES

Situation[a] (P^U - P^L)	Model	Coefficients of Variation							
		Acreage	Production	Price	Stocks	Net Income	Gross Income	Social Cost	Total Loss
	II_M	6.7	10.6	17.4	45.3	44.8	15.7	332.5	72.0
2	II_2	9.2	11.6	2.1	57.1	19.3	10.3	194.9	84.6
(125-120)	II_4	9.4	11.8	1.9	32.0	19.9	10.6	195.6	53.4
	II_6	9.3	11.8	2.0	21.9	19.9	10.6	194.9	38.4
	II_M	4.1	10.6	10.5	11.7	28.7	10.5	143.5	27.1
3	II_2	8.9	11.3	3.8	44.5	17.6	9.2	157.6	75.8
(130-120)	II_4	8.9	11.3	3.8	26.4	17.6	9.2	157.6	52.2
	II_6	8.7	11.1	3.8	18.3	17.6	9.1	150.7	37.4
	II_M	3.8	9.7	10.3	8.2	27.1	10.0	138.1	20.1
4	II_2	8.5	10.9	6.9	34.8	17.5	8.2	112.7	61.1
(140-120)	II_4	8.5	10.9	6.9	21.9	17.5	8.2	112.7	45.6
	II_6	8.1	10.5	7.0	15.2	17.7	8.0	104.1	32.8
	II_M	3.9	9.4	13.4	7.5	27.6	9.5	177.6	30.7
5	II_2	8.2	10.5	9.6	28.1	20.2	8.6	86.9	51.5
(150-120)	II_4	8.2	10.5	9.6	18.0	20.2	8.6	86.9	40.8
	II_6	7.4	9.9	9.8	12.7	20.7	8.3	79.4	29.1
	II_M	3.6	8.4	8.5	46.4	23.3	8.7	255.8	49.1
7	II_2	8.0	10.5	5.3	50.6	17.7	7.9	157.5	73.4
(130-115)	II_4	8.0	10.5	5.3	27.5	17.7	7.9	157.5	46.4
	II_6	8.0	10.5	5.3	18.8	17.7	7.9	157.7	33.9
	II_M	2.5	9.4	7.4	23.4	18.8	6.5	195.4	32.5
8	II_2	7.5	9.9	7.7	39.1	18.4	7.4	140.5	65.2
(140-115)	II_4	7.5	9.9	7.7	22.9	18.4	7.4	140.5	44.3
	II_6	7.5	9.9	7.7	16.1	18.4	7.4	140.5	33.6
	II_M	3.2	10.2	8.9	18.2	22.1	7.7	135.4	25.3
9	II_2	7.2	9.4	10.2	31.3	21.6	8.3	117.5	58.4
(150-115)	II_4	7.2	9.4	10.2	19.4	21.6	8.3	117.5	42.6
	II_6	7.1	9.4	10.2	14.0	21.6	8.3	117.4	33.5

[a]Situation numbers correspond to those given in Table XI.

TABLE XVIII

SELECTED MODEL II SIMULATION RESULTS, NONEQUILIBRIUM SITUATIONS: PERCENT OCCURRENCE OF SIX PRICE-RELATED EVENTS

Situation[a]		Column Number[b]					
(P^U - P^L	Model	1	2	3	4	5	6
	II_M	4	4	34	20	26	16
2	II_2	6	6	30	52	0	12
(125-120)	II_4	0		36	52	0	12
	II_6	0		36	50	2	12
	II_M			24	8	44	24
3	II_2			34	48	0	18
(130-120)	II_4			34	48	0	18
	II_6			34	46	2	18
	II_M			20	10	34	36
4	II_2			30	38	0	32
(140-120)	II_4			30	38	0	32
	II_6			30	36	2	32
	II_M			14	10	40	36
5	II_2			24	30	0	46
(150-120)	II_4			24	30	0	46
	II_6			22	26	4	46
	II_M			26	24	10	40
7	II_2			26	30		44
(130-115)	II_4			26	30		44
	II_6			26	30		44
	II_M			12	14	24	50
8	II_2			22	28		50
(140-115)	II_4			22	28		50
	II_6			22	28		50
	II_M			6	6	30	58
9	II_2			20	24		56
(150-115)	II_4			20	24		56
	II_6			20	24		56

[a]Situation numbers correspond to those given in Table XI.

[b]Column:

Number	Percentage of:
1	Zero Inventory,
2	Price greater than P^U,
3	Price equal to P^U,
4	Price equal to P^L,
5	Price less than P^L,
6	Price between P^U and P^L.

always be the assumed equilibrium quantity of 1550 except when the carryover is zero as noted above. Price is not necessarily established at 120 when the quantity marketed is 1550 because of the random and dynamic characteristics of the demand function, nor does zero carryover require that price be above 120 for the same reason.

The $\theta = 1.0$ situation is a very rigid inventory policy attempting to market a set amount each year at whatever price is established in the market. As such, it may be compared to the also rigid zero price range situation of Model II which attempts to always maintain price at a set level, letting the quantity marketed vary with demand conditions.

In Model II, additions to stock occur when X_t falls below a certain level, dependent on actual demand, and vice versa for withdrawals from stock. Adjustments to reserve stocks are not affected by current period demand conditions in Model III. Additions to stock occur only when the total available supply for period t is greater than for period t-1, and withdrawals when S_t is less than S_{t-1}.[3]

With an inventory policy of this nature there exists a possibility of something akin to cycling or runs in the level of reserves, especially when θ is equal to or nearly equal to unity. To see why this

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[3]Define SA_t to be the adjustment in stocks during period t: $SA_t = C_t - C_{t-1}$. Then,

$$SA_t = C_t - C_{t-1} = \theta(S_t - Q^*) - \theta(S_{t-1} - Q^*)$$

$$SA_t = \theta(S_t - S_{t-1})$$

If $S_t > S_{t-1}$, then $SA_t > 0$ (an addition to stock, $C_t > C_{t-1}$)
If $S_t < S_{t-1}$, then $SA_t < 0$ (a withdrawal, $C_t < C_{t-1}$).

possibility exists, consider a period t with an unusually large production and sizable carryover from period t-1. Assume that this causes S_t to exceed S_{t-1} so that there will be an addition to reserves as explained above ($C_t > C_{t-1}$). If $\theta = 1.0$, C_t will be large, probably causing S_{t+1} to be even greater than S_t resulting in another addition to stocks ($C_{t+1} > C_t$) which may in turn result in S_{t+2} being even larger, etc. Since the quantity marketed is still 1550 (if $\theta = 1$), there is no tendency for demand to change via the lagged variables. It may be difficult for this trend to be broken, but if random events do combine and result in a withdrawal, the reverse of the above procedure occurs and the stocks dissipate.

The possibility of a run or cycle occurring exists only when acreage is determined via market conditions as in situation III_M and not if acreage is dependent on the deviation of actual from desired carryover as in situations III_2, III_4, and III_6. A trend cannot continue long in the latter situations because a large carryover, for example, immediately reduces acreage and production, thus preventing an ever-increasing supply. An upward run can be broken in the market-determined acreage situation by a series of events, depending on the relative strength of the forces which have built up. For example, if demand in period t-1 was unusually high (a random event) so that the 1550 quantity marketed brought a very low price, acreage for period t will fall because of the lagged price coefficient in the acreage determination equation. If the random yield is also sufficiently low, S_t may be less than S_{t-1} (even though C_{t-1} was large) and the run will be reversed.

This problem did occur in the actual simulation for $\theta = 1.0$ in situation III_M. If stocks were limited to a maximum of 1000, the pattern seemed to be for stocks to reach 1000, then fall to zero and remain nearly there. If an upper limit was not placed on stocks, the trend was generally upward and very large stocks accumulated. Because it was felt that this cycling or runs would probably not be allowed to occur with the severity shown in the simulation, the results when θ = 1.0 for situation III_M were considered unrealistic and are not reported here.

Means and coefficients of variation of nine selected variables for Model III are shown in Tables XIX and XX. The mean values shown in Table XIX follow fairly closely to the pattern one would expect them to take. For example, from Figure 7 (Chapter IV), it is apparent that Model III is not likely to give equilibrium results, on the average ($\bar{A}$ = 62, $\bar{P}$ = 120, $\bar{Q}$ = 1550, etc.), except when θ = 1.0. If θ is less than one there is a definite bias toward smaller stocks and toward lower price resulting in larger quantities produced and consumed.[4]

The bias toward smaller stocks for smaller values of θ is easy to see: with the same excess supply conditions, smaller θ requires that a smaller portion be held as reserve stock and a larger portion of the total available supply be marketed.

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[4]For Model III, average quantity marketed and used by the three consuming sectors, $Q = Q_H + Q_F + Q_E$, is the same as average production so Q is not shown in the table. To see why these values are the same, consider S_t as acquisition: $S_t = X_t + C_{t-1}$. But S_t also has a disposal counterpart: $S_t = Q_t + C_t$. Since the S_t values must be equal for each period, they must be equal as totals and means so that:

$$\bar{S} = \bar{X} + \frac{1}{N}\sum_{t=1}^{N} C_{t-1} = \bar{Q} + \frac{1}{N}\sum_{t=1}^{N} C_t$$

$$\bar{X} + \frac{C_o}{N} = \bar{Q} + \frac{C_N}{N} .$$

When N is large, the difference between the terms in N is negligible and $\bar{X} = \bar{Q}$.

TABLE XIX

SELECTED MODEL III SIMULATION RESULTS: MEANS OF NINE VARIABLES

		Means[a]								
θ	Model	Acreage	Production	Supply	Price	Stocks	Net Income	Gross Income	Social Cost	Total Loss
1.00	III_M	--	--	--	--	--	--	--	--	--
	III_2	61.9	1547	1750	121.4	203	641	1878	3.5	33.9
	III_4	62.0	1550	1950	121.3	400	640	1880	2.6	62.6
	III_6	62.0	1550	2150	121.3	600	640	1880	2.6	92.5
.90	III_M	62.4	1560	1862	120.8	303	634	1882	11.8	57.2
	III_2	62.7	1567	1750	120.4	183	631	1884	4.4	31.8
	III_4	63.6	1590	1950	119.2	360	622	1893	4.8	58.8
	III_6	64.4	1610	2150	118.2	540	613	1901	7.4	88.4
.80	III_M	62.6	1563	1728	121.0	165	635	1887	14.4	39.1
	III_2	63.5	1587	1750	119.4	163	621	1891	7.0	31.5
	III_4	65.2	1630	1950	117.2	320	603	1907	11.9	59.9
	III_6	66.8	1670	2150	115.2	480	585	1920	22.4	94.4
.75	III_M	62.0	1550	1662	121.0	112	610	1850	16.7	33.5
	III_2	63.9	1597	1750	118.9	152	616	1894	9.0	32.0
	III_4	66.0	1650	1950	116.2	300	593	1913	17.4	62.4
	III_6	68.0	1700	2150	113.8	450	570	1930	33.8	101.3
.70	III_M	62.5	1558	1656	121.0	99	625	1875	18.8	33.6
	III_2	64.3	1607	1750	118.4	142	612	1897	11.4	32.8
	III_4	66.8	1670	1950	115.3	280	584	1919	24.0	66.0
	III_6	69.2	1730	2150	112.4	420	555	1938	47.9	110.9

[a]Units correspond to those given in footnote 1 of Chapter IV.

TABLE XX

SELECTED MODEL III SIMULATION RESULTS: COEFFICIENTS OF VARIATION OF NINE VARIABLES

θ	Model	Coefficients of Variation								
		Acreage	Production	Supply	Price	Stocks	Net Income	Gross Income	Social Cost	Total Loss
1.00	III_M	--	--	--	--	--	--	--	--	--
	III_2	7.9	11.1	7.4	12.6	59.1	43.9	16.6	341.9	59.8
	III_4	8.3	11.4	6.7	12.5	31.5	44.9	17.0	158.8	31.2
	III_6	8.3	11.4	6.1	12.5	21.0	44.9	17.0	158.8	21.2
.90	III_M	6.4	10.6	20.4	13.5	104.8	41.7	16.0	273.5	16.0
	III_2	7.2	10.6	7.4	12.5	59.6	42.0	15.3	293.9	15.3
	III_4	7.6	10.9	6.8	12.0	32.3	42.9	15.5	107.6	15.5
	III_6	7.6	10.9	6.2	11.8	21.8	43.1	15.3	94.0	15.3
.80	III_M	5.7	9.9	14.5	13.5	105.1	38.3	14.2	270.7	14.2
	III_2	6.4	10.2	7.5	12.6	60.2	40.5	14.2	218.3	14.2
	III_4	6.8	10.4	7.0	11.8	33.1	41.5	14.2	94.1	14.2
	III_6	6.8	10.3	6.4	11.5	22.5	42.1	14.0	73.7	14.0
.75	III_M	5.5	10.2	12.9	14.0	114.6	37.5	13.5	225.6	13.5
	III_2	6.4	10.2	7.6	12.7	60.5	40.0	13.8	189.2	13.8
	III_4	6.4	10.1	7.1	11.8	33.5	41.0	13.7	89.5	13.7
	III_6	6.3	10.1	6.6	11.5	23.0	42.0	13.4	68.3	13.4
.70	III_M	5.8	10.1	12.1	14.3	112.3	37.0	13.5	202.4	13.5
	III_2	5.7	9.7	7.6	12.8	60.7	39.6	13.4	166.2	13.4
	III_4	6.1	9.9	7.1	11.9	33.9	40.7	13.2	86.2	13.2
	III_6	5.9	9.9	6.7	11.5	23.4	42.1	12.9	64.9	12.9

Price is biased downward because with average supply conditions, S_t will usually exceed 1550. With normal demand conditions, this must result in price less than 120 when θ is less than one. This lower price results in quantity demanded increasing via the negative coefficient for P_t in the demand function. This effect is partially offset in the longer run as demand increases via the positive coefficient for QE_{t-1}.

These effects require a sort of cooperation from supply so that the method of acreage determination affects which forces prevail. When acreage is tied to the deviation of actual from desired carryover, smaller θ resulting in low stock means that this deviation is most often positive and acreage and production are high. S_t will, on the average, be sizable and price lower as previously discussed.

In the actual simulation for situations III_2, III_4, and III_6, price fell substantially as θ decreased, indicating that the negative P_t coefficient coupled with greater production prevailed over the offsetting influence of an actually greater demand which would tend to keep price up. The high acreage and low price conditions are naturally more severe for the higher desired carryover situations because the deviation of actual from desired carryover is greater.

When acreage is determined according to market conditions via equation (7) of Chapter IV as in situation III_M, the effects are somewhat different. Given that $\theta < 1$ causes a low price, according to the equation, acreage the next period will decline causing production and total supply to be less. In the actual simulation, this pressure seemed to be enough to keep price and acreage nearly the same for all values of θ. However, the low values for S_t associated with small values of θ meant that stocks were usually small and often zero. This is shown in Table XXI which gives the percentage occurrence of zero inventory for the 20 simulated situations of Model III.

TABLE XXI

SELECTED MODEL III SIMULATION RESULTS:
PERCENT OCCURRENCE OF ZERO INVENTORY

θ	Model			
	III_M	III_2	III_4	III_6
1.00	--	6.4	0	0
.90	21.6	6.5	0	0
.80	25.8	6.5	0	0
.75	32.2	6.5	0	0
.70	31.5	6.5	0	0

Looking at the income values in Table XIX for the managed-supply situations III_2, III_4, and III_6 shows that gross income is higher for smaller values of θ, indicating that high production more than offsets low price for Model III. But smaller values for θ result in lower net incomes due to the increased costs associated with larger acreages. Gross incomes are also higher when the desired carryover is larger, again indicating that the production component of total revenue outweighs the price component. And again, when the costs of producing additional acres are subtracted, net incomes are lower for the higher target carryover situations.

For the free market acreage situation III_M, the comparatively lower acreage (for each value of $\theta < 1$) gives lower gross income than for situations III_2, III_4, and III_6 in spite of the fact that for situation III price does not decrease with θ as it does in the managed-supply situations. This again shows the dominance of the production component in gross income. The lower acreage figures for situation III_M result in production costs which are enough lower to cause net income to be greater than for III_2, III_4, and III_6.

The results pertaining to income may be summarized as follows:

1. High net income is associated with:
    a. θ large (close to 1.0),
    b. C^* small.
2. High gross income is associated with:
    a. θ small,
    b. C^* large.
3. Gross income is greater for situation III_M.
4. Net income is lower for situation III_M.

Table XIX shows that average social cost for all four situations of Model III is higher for smaller values of θ. This result follows from the fact that smaller θ values cause: 1) less storage, and 2) the quantity marketed to be further removed from the assumed equilibrium for the period. Less storage results in lower storage costs for decreasing values of θ since storage cost is proportional to the storage level. Both of these trends are consistent with expected results based on explanations and definitions already given.

As was mentioned at the beginning of this section, Model III was designed especially to test the results of the dynamic programming analysis of Chapter III. Results presented there indicated that according to the dynamic programming inventory model, total discounted expected losses over an infinite planning horizon are minimized when approximately 85 percent of the excess supply is treated as carryover. For this to hold in the simulation analysis requires that average total loss value be least for θ between .90 and .80.

Figure 9 shows the relationships among social cost, storage cost and total loss for the four situations of Model III. In all four cases (and for θ decreasing), the decreasing storage cost and increasing social cost functions combine to give a total loss function that declines to a minimum, then increases. Since storage costs are nearly linearly decreasing, the U-shaped nature of total loss must come mostly from social cost which appears to increase at an increasing rate.

Table XIX and Figure 9 also show that both social costs and storage costs are higher for larger values of C^* with the absolute differences greater for storage than social costs. This gives rise to total loss values much higher for the larger values of C^*. The total loss for situation III_M falls generally between those of III_2 and III_4.

The minimum points on the total loss functions for the three situations where acreage is tied to carryover seem to come quite close to the 85 percent point. The minimum for situation III_M is nearer 75 percent. In situation III_M, storage cost decreases more rapidly and social cost increases less rapidly than for situations III_2, III_4, and III_6. Both of these conditions cause the minimum of total loss to be associated with smaller θ values, and both conditions result from the dynamic

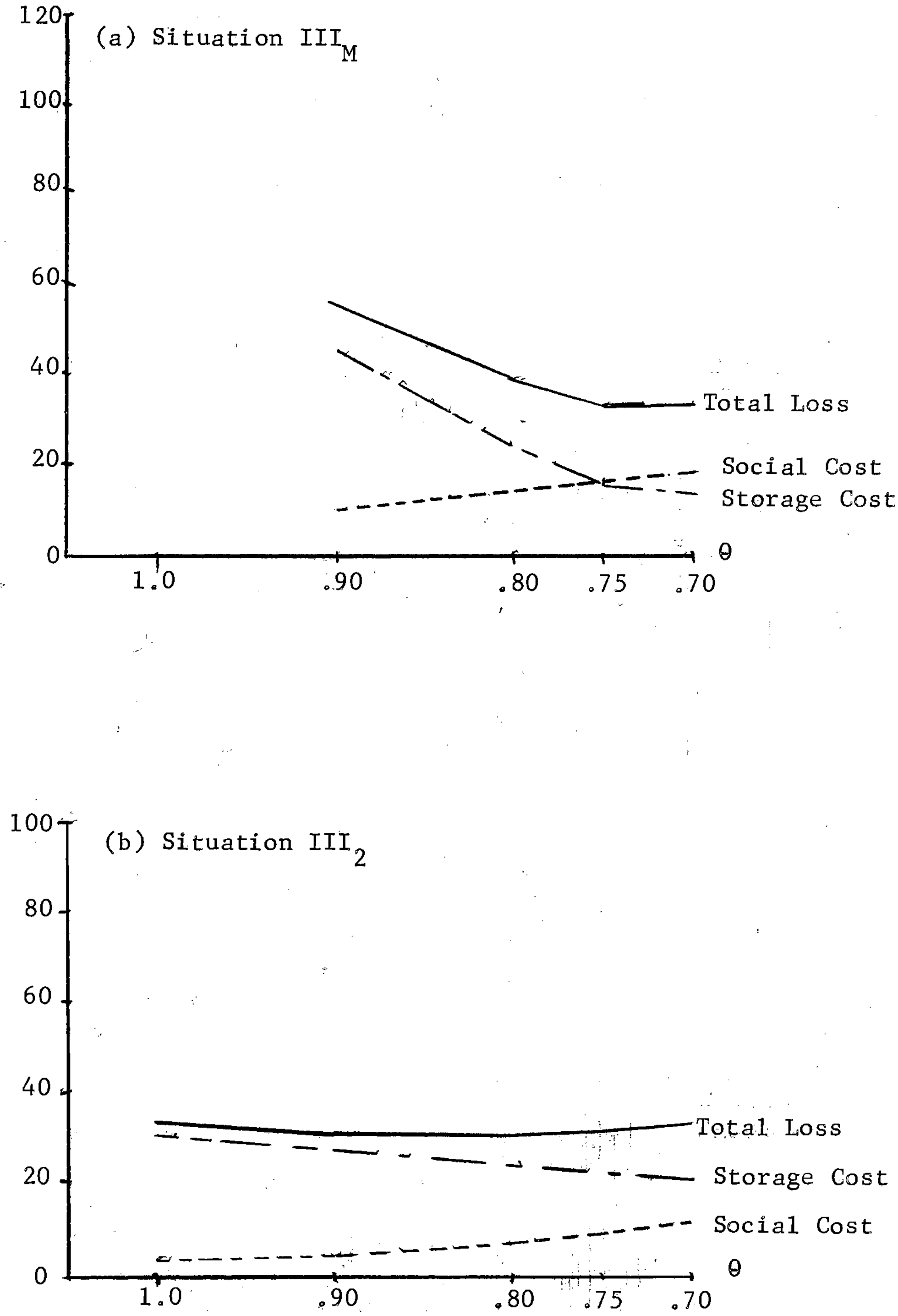
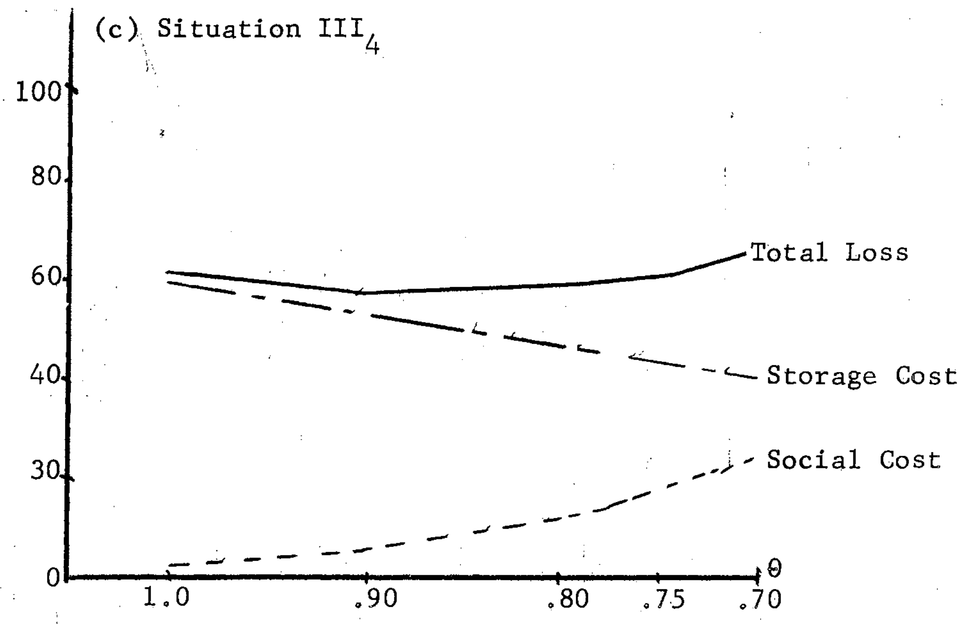


Figure 9. Social Cost, Storage Cost, and Total Social Loss, Model III



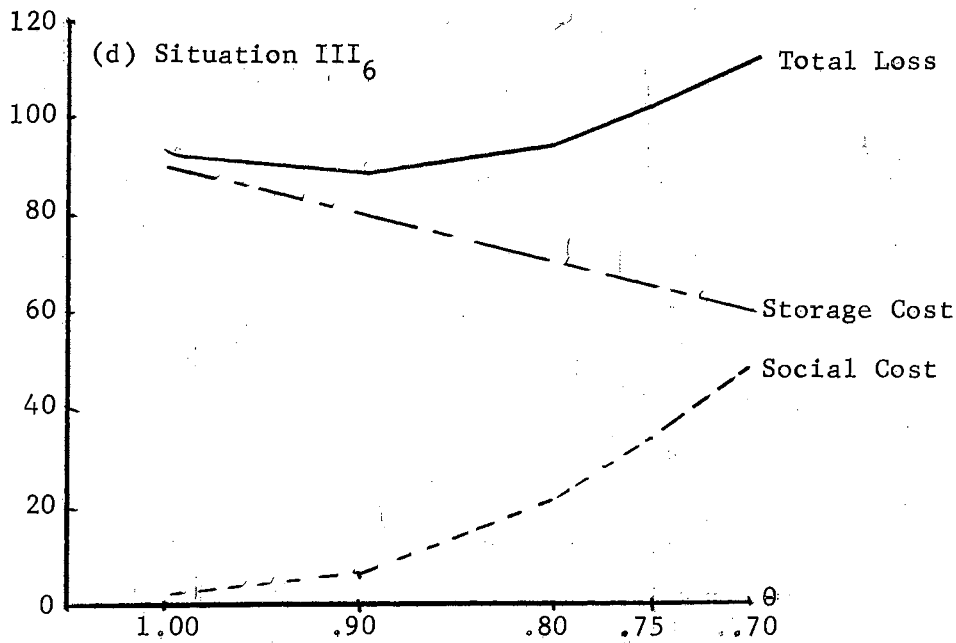


Figure 9. (Continued)

interactions of jointly determined demand and supply through the acreage equation.

Previous discussion has mentioned that a weakness of the dynamic programming inventory model, as formulated, is that it does not allow the acreage decision to be incorporated into the model. To this extent, the simulation results should be considered an improvement of the dynamic programming results. This indicates that, speaking from a total social loss point of view, the 85 percent value is too high and should be closer to 75 percent if acreage determination is left to market forces.

To summarize the social cost, storage cost and total loss relationship:

1. As θ decreases:
    a. Social cost increases,
    b. Storage cost decreases,
    c. Total social loss first decreases, then increases with the minimum point being fairly close to θ = .85 for situations III_2, III_4, and III_6 and near .75 for situation III_M.
2. As C^* increases:
    a. Social cost, storage cost and total loss increase.

With regard to the stability conditions of situation III_M, when acreage is market determined lower values of θ give slightly more stable acreage and production conditions while reserve stocks show more variability as measured by the coefficients of variation.[5] For the variables for which stability is probably most important, namely the price and income series, lower values of θ cause price to be slightly more variable but incomes more stable. When acreage is determined as in situations III_2, III_4, and III_6, price is also slightly more stable for smaller θ values. The other trends are nearly the same as for situation III_M.

Within the three managed-supply situations, when C^*, the target or desired carryover, is large, price is slightly less variable and net income slightly more variable than when C^* is small. For the other variables for which stability is important, there are no noticeable differences among the three C^* situations.

It is seen that net income is less variable and acreage slightly more so while stock variability is much greater and price variability slightly greater for situation III_M than for III_2, III_4, and III_6.

The results of Model III with respect to stability conditions may be summarized as follows:

1. Smaller θ values are associated with:
    a. More stable acreage, production, and incomes,
    b. Slightly more stable price for situation III_M, slightly less stable price for situations III_2, III_4, and III_6.
2. Larger C^* values are associated with:
    a. Slightly more stable price,
    b. Slightly less stable income.

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[5]Variability as measured by the standard deviation shows that lower θ values are associated with more stable stock levels, just the opposite trend from the coefficient of variation measure.

3. The market-determined acreage situation gives:

    a. Slightly more stable incomes and acreage,

    b. Much less stable reserve stocks and slightly less stable price.

Comparison of Models I, II, and III

The task of this section is to compare the various reserve management policies that have been discussed to this point; that is, to compare the results from the simulation of Models I, II, and III. The three preceding sections have given comparisons internally within each model, briefly discussing selected results of 72 different situations. These sections have also sometimes explained the interrelationships within each model that caused the results shown -- whenever it was possible and deemed necessary. This section will merely point out the differences that exist between models without again explaining why.

Because of the great number of situations, the models will be compared on a limited number of bases -- hopefully those that are most economically significant. Variables whose levels will be presented include price, net and gross income, reserve stocks, social cost and total loss. For comparison of stability, the coefficients of variation of six variables are shown: acreage, production, price, net and gross income, and reserve stocks. Each of these variables and measures have some significance or bearing on relative performance of the three models. Also shown is the performance of each model with respect to the likelihood of inadequate reserves -- zero inventory.

The Free Market Acreage Situations

Looking first only at the situations where acreage is determined according to market conditions as in equation (7) of Chapter IV,[6] some of the variables show sizable differences among the means (Table XXII) while for other variables there are hardly any differences. Also, sometimes there is more variation within model situations and sometimes more between. Where differences exist, each of the 18 situations show both "desirable" and "undesirable" results: no model scores consistently above the others in terms of desirable characteristics for all variables.

For the income variables alone, no model is consistently best. Both the highest and lowest net and gross incomes are associated with disequilibrium situations of Model II, but if the disequilibrium situations are not considered, both the highest and lowest incomes are found in Model III. The ranges within the income series are about the same for Models II and III with I falling within the ranges for both II and III.

Social cost for I also falls within the ranges of both II and III with the highest social cost coming in II when the price spread is widest and in III when θ is largest. When storage costs are added to

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[6]When the purpose is to evaluate overall reserve management policies, it should not be necessary to compare the two supply determination methods. The two methods represent entirely different philosophies: supply determination is either left to market forces or it is not. There is no reason to presume that a particular management policy that is effective in one instance is necessarily either effective or ineffective in another, nor is the supply management philosophy assumed in this study the only one possible. The results obviously apply only to the rules and situations presented here, but others could be analyzed separately.

TABLE XXII

COMPARISONS AMONG MODELS I, II, AND III, ACREAGE MARKET-DETERMINED

		Means[a]						Coefficients of Variation					
Model		Price	Net Income	Gross Income	Social Cost	Total Loss	Stocks	Acres	Production	Price	Net Income	Gross Income	Stocks
I		120.8	627	1875	17.3	76.8	397	5.2	9.4	11.7	26.9	10.4	14.5
	Situation[b]												
	1	120.5	629	1875	21.3	105.2	559	5.0	9.8	10.8	34.5	13.0	61.2
	6	120.3	620	1863	16.5	97.7	541	4.6	9.4	9.6	28.8	11.0	65.6
II	10	120.2	617	1859	15.8	100.8	567	4.2	9.2	9.4	26.2	10.0	59.7
	11	120.9	624	1874	28.6	63.4	232	5.5	9.8	14.5	34.1	12.1	96.8
	12	121.6	633	1890	41.1	53.0	79	6.0	10.0	17.8	41.1	13.8	140.4
	2	122.5	651	1919	31.1	135.8	698	6.7	10.6	17.4	44.8	15.7	45.3
	3	118.0	577	1797	26.5	166.6	934	4.1	10.6	10.5	28.7	10.5	11.7
II	4	122.8	665	1936	22.4	163.9	943	3.8	9.7	10.3	27.1	10.0	8.2
	5	121.6	664	1927	28.0	172.7	965	3.9	9.4	13.4	27.6	9.5	7.5
	7	120.5	626	1870	15.7	136.9	808	3.6	8.4	8.5	23.3	8.7	46.4
	8	120.4	629	1876	17.9	146.1	855	2.5	9.4	7.4	18.8	6.5	23.4
	9	122.0	652	1918	21.1	165.4	962	3.2	10.2	8.9	22.1	7.7	18.2
	θ												
	1.00	--	--	--	--	--	--	--	--	--	--	--	--
	.90	120.8	634	1882	11.8	57.2	303	6.4	10.6	13.5	41.7	16.0	104.8
III	.80	121.0	635	1887	14.4	39.1	165	5.7	9.9	13.5	38.3	14.2	105.1
	.75	120.0	610	1850	16.7	33.5	112	5.5	10.2	14.0	37.5	13.5	114.6
	.70	121.0	625	1875	18.8	33.6	99	5.8	10.1	14.3	37.0	13.5	112.3

[a]Units correspond to those given in footnote 1 of Chapter IV.

[b]Situation numbers correspond to those given in Table XI.

social costs to give total social losses, Model III values are clearly lowest, and highest in the disequilibrium situations of II because of the accumulation of stocks. Total loss for I is within the range of the equilibrium situation of II. The advantages of low total loss for Model III, due primarily to low storage costs, are largely offset by the high likelihood of reserve stocks being inadequate as shown in Table XXIII.

The coefficients of variation given in Table XXII show that Model III provides generally less stable conditions for all variables except stocks than do Models I and II. The coefficients of variation for Model I fall within the ranges of those for II in all cases except for stocks, in which case I is much lower. It is interesting to note that if the disequilibrium situations of Model II are not considered, situation 10 of Model II (when $P^U = 130$, $P^L = 110$) gives the greatest stability for all variables except stocks. When the disequilibrium cases are considered, greatest stability is achieved for all variables except production and stocks when P^U is 140 and P^L is 115 as in situation 8 of Model II.

Another way to compare the three models is to look at the sacrifices and gains from choosing I, say, compared to a situation subjectively selected from II or III as "best."

Levels or means must be used to select a best θ value because the coefficients of variation for III are not sensitive to θ. The situation where $\theta = .80$ has an advantage based on a subjective ranking: $\theta = .80$ is preferred to $\theta = .75$ on the basis of levels of social cost, net income and gross income, and to $\theta = .90$ for gross income and total loss. Then comparing I to III with $\theta = .80$ shows I with more desirable

TABLE XXIII

PERCENT OCCURRENCE OF ZERO INVENTORY, MODELS I, II, AND III

		Supply Situation			
Model		MKT	200	400	600
I		0.	0.	0.	0.
	Situation				
II	1	8.6	12.6	2.4	0
	6	9.3	9.4	1.5	0
	10	5.8	6.0	1.3	0
	11	19.1	7.4	1.1	0
	12	37.1	7.9	0	0
	θ				
III	1.00	--	--	--	--
	.90	21.6	6.4	0	0
	.80	25.8	6.5	0	0
	.75	37.2	6.5	0	0
	.70	31.5	6.5	0	0
	Situation				
II	2	4.0	6.0	0	0
	3	0	0	0	0
	4	0	0	0	0
	5	0	0	0	0
	7	0	0	0	0
	8	0	0	0	0
	9	0	0	0	0

income stability but slightly less desirable income levels, greater social cost and much greater total loss (because stocks are small and often zero for Model III).

A subjective ranking could select situation 10 as being the "best" of Model II situations based on overall low variability and low social cost even though it shows neither best nor worst income or total loss levels. Comparing I with II, situation 10, shows the latter slightly more stable, except for stocks, and having more desirable income levels but less desirable social cost and total loss levels. Comparing II_{10} to III, $\theta = .80$, shows II more stable but with lower net and gross incomes and higher total social loss.

The Controlled Acreage Situations

Comparisons among the means of the six variables for the three major models show some interesting results when acreage is set semi-endogenously by aiming for a particular carryover level each period (Tables XXIV, XXV, and XXVI). It is quite clear that most of the "desirable" results are associated with Model III and most of the undesirable results are associated with Model II -- when the disequilibrium situations are not considered.[7, 8] The most desirable results (high price, net income, and gross income and low social cost and total loss) of Model III are generally associated with the higher values of

[7]Because these disequilibrium situations were simulated for only 55 periods, it is possible that the resulting values are not expected values so that comparisons to other situations might not be valid.

[8]It must be assumed that there is not actually a "desirable" stock level: the purpose of the analysis is to determine how much stock and how much to manage these stocks so that the average stock level itself is incidental.

TABLE XXIV

COMPARISONS AMONG MODELS I, II, AND III, DESIRED CARRYOVER 200 MILLION BUSHELS

Model		Means[a]						Coefficients of Variation					
		Price	Net Income	Gross Income	Social Cost	Total Loss	Stocks	Acres	Production	Price	Net Income	Gross Income	Stocks
I		129.6	683	1794	41.9	96.0	361	4.4	9.5	11.9	26.8	11.8	17.3
	Situation[b]												
	1	121.6	626	1827	24.2	61.6	243	12.2	14.6	5.9	27.7	15.4	73.9
	6	121.3	619	1824	24.1	60.7	243	11.0	13.7	6.8	23.7	12.9	67.9
II	10	121.2	614	1820	27.2	63.6	243	9.8	12.8	8.5	22.9	11.3	60.8
	11	119.3	602	1855	32.2	59.7	183	7.8	11.3	12.3	28.4	10.9	66.6
	12	116.7	581	1884	45.3	63.7	122	5.6	9.9	14.9	35.4	11.5	75.0
	2	122.6	631	1839	13.8	50.1	242	9.2	11.6	2.1	19.3	10.3	57.1
	3	124.4	640	1809	22.8	66.5	291	8.9	11.3	3.8	17.6	9.2	44.5
	4	128.1	665	1791	36.7	88.5	344	8.5	10.9	6.9	17.5	8.2	34.6
	5	131.5	684	1771	55.2	114.3	394	8.2	10.5	9.6	20.2	8.6	28.1
II	7	122.1	627	1837	15.9	51.7	238	8.0	10.5	5.3	17.7	7.9	50.6
	8	124.9	647	1823	21.5	63.8	282	7.5	9.9	7.7	18.4	7.4	39.1
	9	127.8	666	1807	32.1	80.9	325	7.2	9.4	10.2	21.6	8.3	31.3
	θ												
	1.00	121.4	641	1878	3.5	33.9	203	7.9	11.1	12.6	43.9	16.6	59.1
	.90	120.4	631	1884	4.4	31.8	183	7.2	10.6	12.5	42.0	15.3	59.6
	.80	119.4	621	1891	7.0	31.5	163	6.4	10.2	12.6	40.5	14.2	60.2
III	.75	118.9	616	1894	9.0	32.0	152	6.0	9.9	12.7	40.0	13.8	60.5
	.70	118.4	612	1897	11.4	32.8	142	5.7	9.7	12.8	39.6	13.4	60.7

[a]Units correspond to those given in footnote 1 of Chapter IV.

[b]Situation numbers correspond to those given in Table XI.

TABLE XXV

COMPARISONS AMONG MODELS I, II, AND III, DESIRED CARRYOVER 400 MILLION BUSHELS

Model		Means[a]						Coefficients of Variation					
		Price	Net Income	Gross Income	Social Cost	Total Loss	Stocks	Acres	Production	Price	Net Income	Gross Income	Stocks
I		120.9	625	1868	15.0	74.5	396	3.7	9.0	11.9	28.8	11.1	14.8
	Situation[b]												
	1	120.2	614	1834	26.3	90.0	425	13.6	15.7	1.2	27.0	15.7	48.9
	6	120.2	609	1828	25.1	88.9	426	12.1	14.5	3.8	21.8	12.8	43.4
	10	120.4	606	1822	27.0	91.6	430	10.5	13.3	6.8	21.0	11.0	37.4
II	11	118.5	594	1856	33.2	88.8	371	8.3	11.7	11.2	26.4	10.2	35.8
	12	115.7	572	1887	49.9	95.7	305	6.3	10.3	14.0	33.9	10.9	34.5
	2	122.1	630	1840	13.8	79.7	440	9.4	11.8	1.9	19.9	10.6	32.0
	3	124.4	640	1809	27.8	96.5	491	8.9	11.3	3.8	17.6	9.2	26.4
	4	128.1	665	1791	36.7	118.4	544	8.5	10.9	6.9	17.5	8.2	21.9
II	5	131.5	684	1171	55.2	144.3	593	8.2	10.5	9.6	20.2	8.6	18.6
	7	122.1	627	1837	15.9	81.7	438	8.0	10.5	5.3	17.7	7.9	27.5
	8	124.9	647	1323	21.5	93.8	482	7.5	9.9	7.7	18.4	7.4	22.9
	9	127.8	666	1807	32.1	110.9	525	7.2	9.4	10.2	21.6	8.3	19.4
	θ												
	1.00	121.3	640	1880	2.6	62.6	400	8.3	11.4	12.5	44.9	17.0	31.5
	.90	119.2	622	1893	4.8	58.8	360	7.6	10.9	12.0	42.9	15.5	32.3
III	.80	117.2	603	1907	11.9	59.9	328	6.8	10.4	11.8	41.5	14.2	33.1
	.75	116.2	593	1913	17.4	62.4	300	6.4	10.1	11.8	41.0	13.7	33.5
	.70	115.3	584	1919	24.0	66.0	280	6.1	9.9	11.9	40.7	13.2	33.9

[a]Units correspond to those given in footnote 1 of Chapter IV.

[b]Situation numbers correspond to those given in Table XI.

TABLE XXVI

COMPARISONS AMONG MODELS I, II, AND III, DESIRED CARRYOVER 600 MILLION BUSHELS

Model		Means[a]						Coefficients of Variation					
		Price	Net Income	Gross Income	Social Cost	Total Loss	Stocks	Acres	Production	Price	Net Income	Gross Income	Stocks
I		113.0	558	1936	55.1	119.4	428	3.1	8.5	11.5	30.8	10.1	12.3
	Situation[b]												
	1	119.8	612	1838	25.9	118.5	617	13.5	15.6	1.1	27.3	15.7	33.6
	6	120.0	608	1830	24.6	117.9	622	11.9	14.4	3.8	21.9	12.7	29.5
II	10	120.4	606	1822	27.0	121.5	630	10.5	13.3	6.8	21.0	11.0	25.5
	11	118.5	595	1859	33.7	119.2	570	8.4	11.7	11.3	26.4	10.3	23.6
	12	115.6	570	1889	52.0	127.3	502	6.6	10.5	14.1	34.0	10.9	21.9
	2	122.1	630	1840	13.6	109.6	640	9.3	11.8	2.0	19.9	10.6	21.9
	3	124.3	640	1810	21.9	125.4	689	8.7	11.1	3.8	17.6	9.0	18.3
	4	128.0	665	1794	34.8	145.9	741	8.1	10.5	7.0	17.7	8.0	15.2
II	5	131.2	684	1775	51.6	169.9	758	7.4	9.9	9.8	20.7	8.3	12.7
	7	122.1	627	1837	15.9	111.7	638	8.0	10.5	5.3	17.7	7.9	18.8
	8	124.9	647	1823	21.5	123.8	682	7.5	9.9	7.7	18.4	7.4	16.0
	9	127.8	666	1807	32.1	140.9	725	7.1	9.4	10.2	21.6	8.3	14.0
	θ												
	1.00	121.3	640	1880	2.6	92.5	600	8.3	11.4	12.5	44.9	17.0	21.0
	.90	118.2	613	1901	7.4	88.4	540	7.6	10.9	11.8	43.1	15.3	21.8
	.80	115.2	585	1920	22.4	94.4	480	6.8	10.3	11.5	42.1	14.0	22.5
III	.75	113.0	570	1930	33.8	101.3	450	6.3	10.1	11.5	42.0	13.4	23.0
	.70	112.4	555	1938	47.9	110.9	420	5.9	9.9	11.5	42.1	12.9	23.4

[a]Units correspond to those given in footnote 1 of Chapter IV.

[b]Situation numbers correspond to those given in Table XI.

θ: θ = 1.0 or .90, or .80 in the case of total loss. The one exception to this is gross income which is highest in all three cases when θ is .70, the smallest value. The only exception to the desirability of Model III -- if desirability of a model is measured by a narrow criterion -- is that when C^* is 200, Model I gives the greatest average price and net income (but at the same time results in the lowest gross income and total loss). The undesirable results associated with Model II come when the price spread is widest -- $P^U = 150$, $P^L = 90$.

Considerations of Model II's disequilibrium situations show that situation 5 has nearly all of both the most and least desirable properties. This is the situation where there is a wide upper and a zero lower price range ($P^U = 150$, $P^L = 120$) around the desired price so that it is relatively easier, according to the rule being followed, to add to than to withdraw from stocks. This one situation consistently gives highest prices and net incomes, but also the lowest or nearly lowest gross incomes, the highest or nearly highest social cost, and always the highest total losses by a wide margin. These results are not unexpected based on the earlier discussion of Model II.

The coefficients of variation as indicators of stability conditions for the three models when acreage is tied to carryover are given in Tables XXIV, XXV, and XXVI. Although relative stability does not seem to be influenced by the value of C^*, the general stability conditions are considerably different than when acreage is market-determined. For example, under the market-determined acreage condition, situation 10 of Model II gives the lowest coefficient of variation for five of the six variables considered, but shows very high stability only for the income variables when acreage is set outside the market (again ignoring the disequilibrium situations of Model II).

Model III shows relatively very unstable income conditions, especially for the $\theta = 1.0$ situation. It should be noted that this is the same situation for which level of net income and social cost are most desirable.

Model I shows a clearly more stable production situation -- the coefficients of variation for acreage and production being significantly lower than for II and III. This may or may not be desirable. If producers are able to make adjustments in acreage and production without extraordinary expenses or without serious loss of total income for their entire operation, sizable production adjustments may not be undesirable. If the reserve management policy puts a high premium on production stability, there may be a sacrifice in the level or stability of income or the ability of the program to maintain adequate reserves.

As can be expected, price is both most stable and unstable for Model II, with the greatest variability associated with the large price spread situations. Models I and III are similar with regard to price stability, and generally about the same as the wide price range situations of Model II. It should be noted that the operation of Model II represents a "managed stock policy" in which private trade groups carry no stock for profit. However, private interests probably would actually perform stocking operations for those situations in which the price spread is wide. The fact that Model II as constructed does not consider the effects of private stocking operations undoubtedly means that the variability of the price and income series for the wide price spread situations are overstated here.

It is again possible to subjectively select a situation from Models II and III for comparisons with I. For III, the situation when θ is .90 shows reasonably good characteristics, generally representing a compromise between more desirable mean values of income and social cost but less desirable stability than for $\theta = 1.0$ and the opposite for $\theta = .80$. The same is accomplished by choosing situation 10 of Model II. Stability is generally good and sacrifices in income levels, social cost and total loss are not great. In addition, situation 10 shows least likelihood of zero inventory when C^* is 200.

Then comparing I with II, situation 10, shows I preferred for net income level and a more stable production situation. Situation II_{10} shows a higher gross income, considerably lower social cost and total loss, and slightly more stable net income. Comparing I with III, $\theta = .90$, shows I with a more stable production situation and greater and more stable net income. III has preferred gross income and much lower social cost and total loss. Other differences are not great. The same comparisons between II and III shows III to be preferred in all cases except that price and income are considerably less stable than for II.

Figures 10 and 11 summarize graphically much of the information presented in this section. From these figures it is much easier to see the magnitudes of the differences that exist among the three models and among situations within each. For Figure 10, a particular horizontal line covers the range of mean values of all situations for the variable and model indicated. For example, the mean price values resulting from all five choices of θ fall somewhere between 117.5 and 121.5 (Figure 10(a)). Since Model I has only one situation, the mean

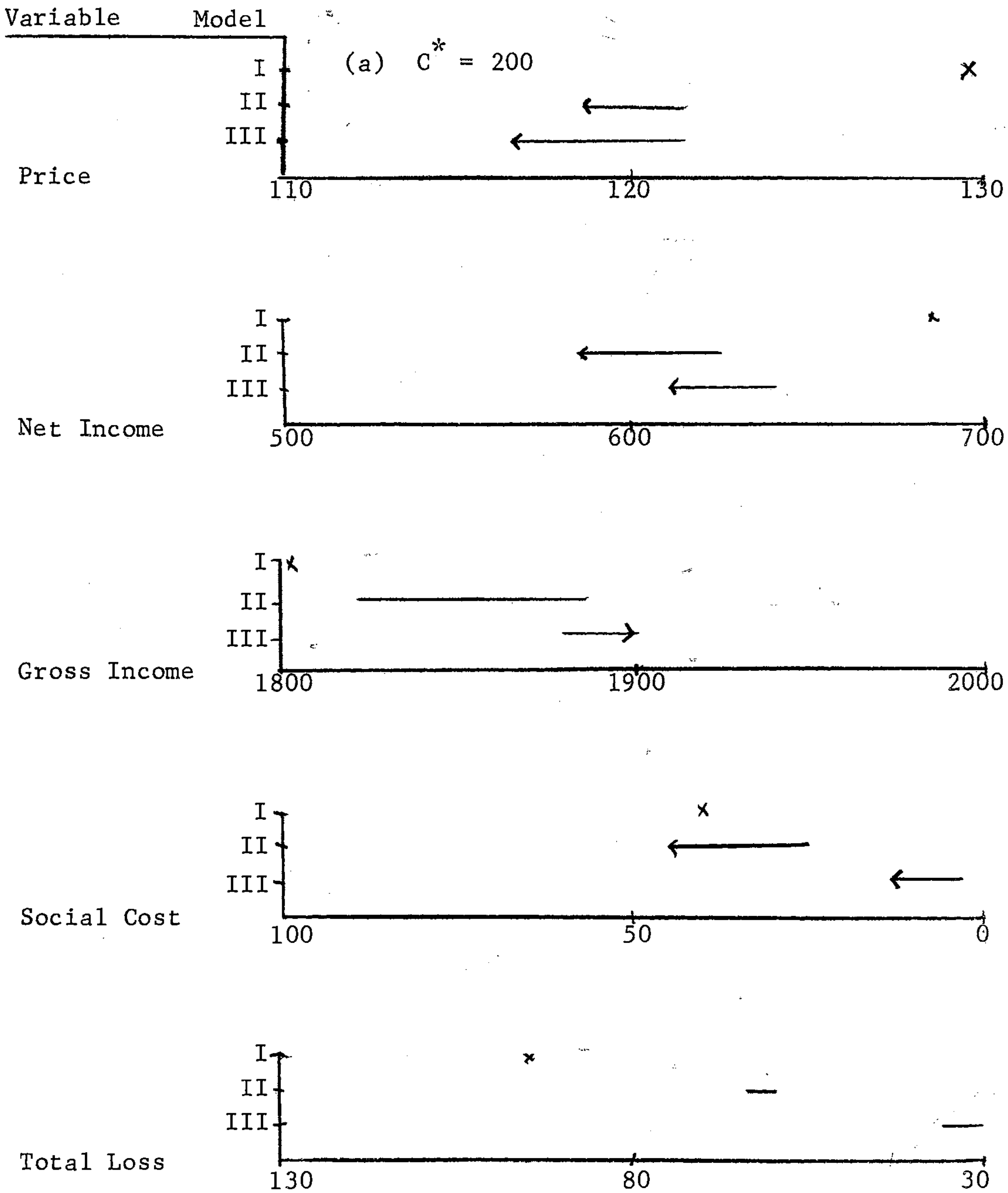


Figure 10. Ranges in Means of Five Variables, Models I, II, and III

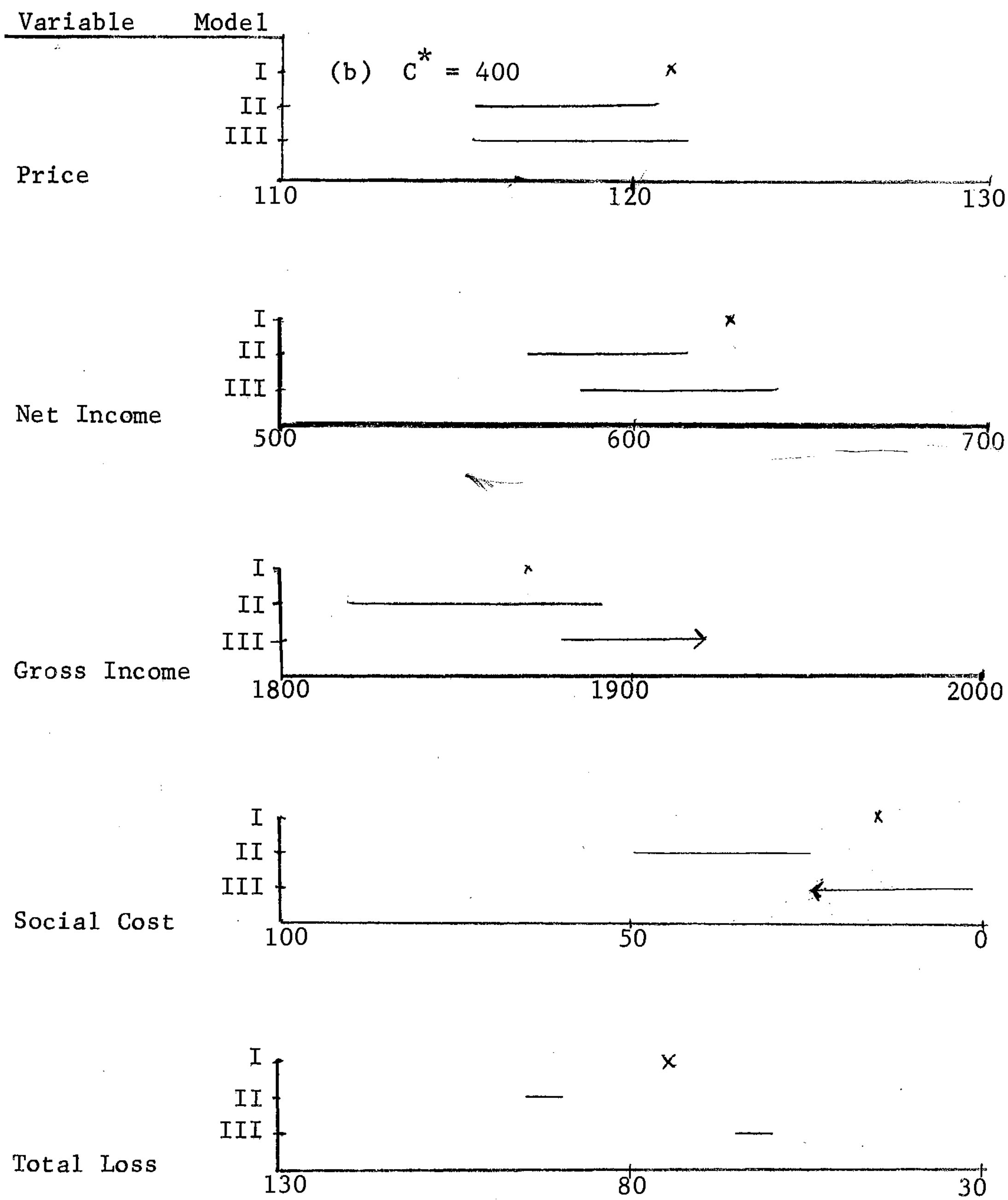


Figure 10. (Continued)

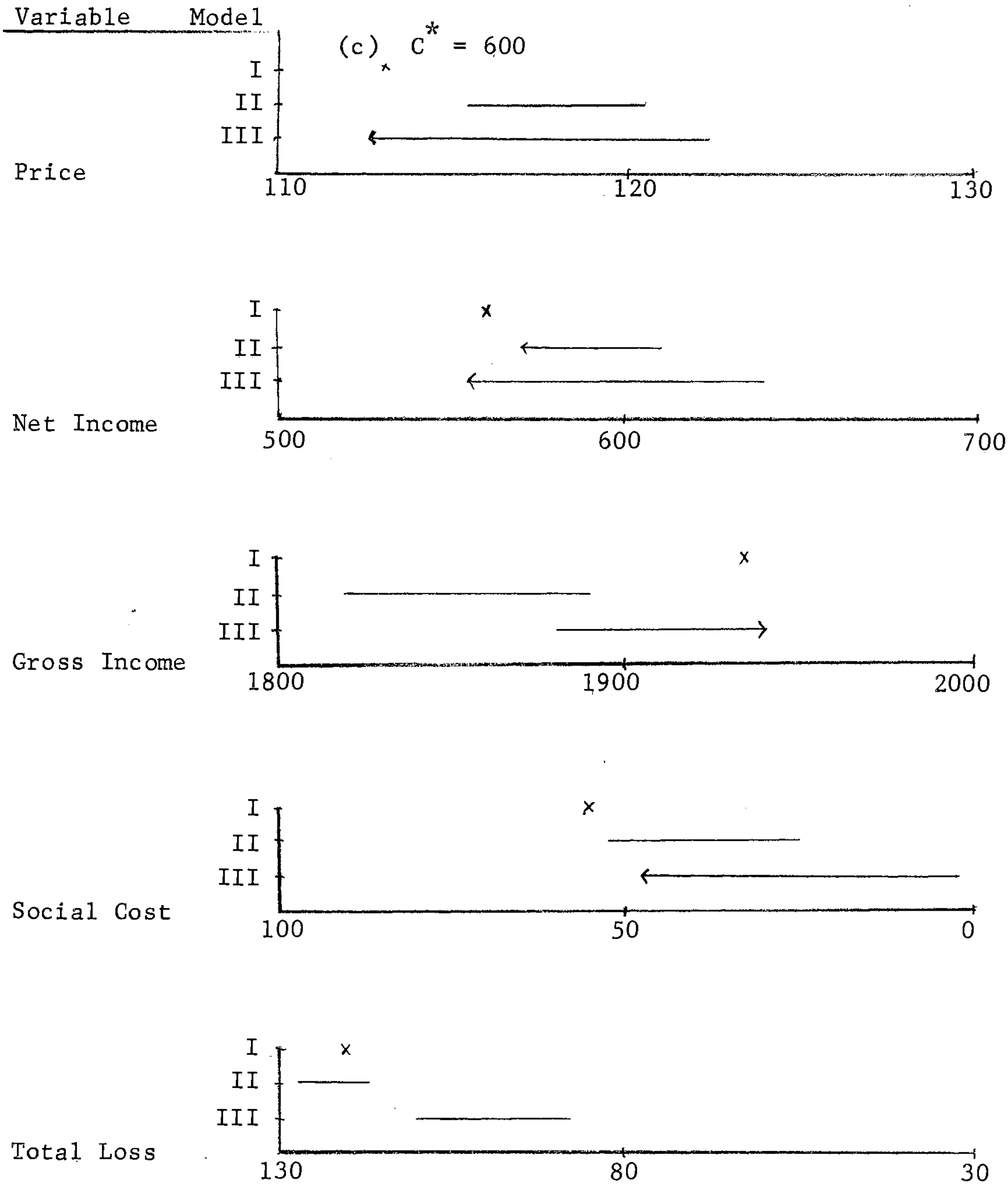


Figure 10. (Continued)

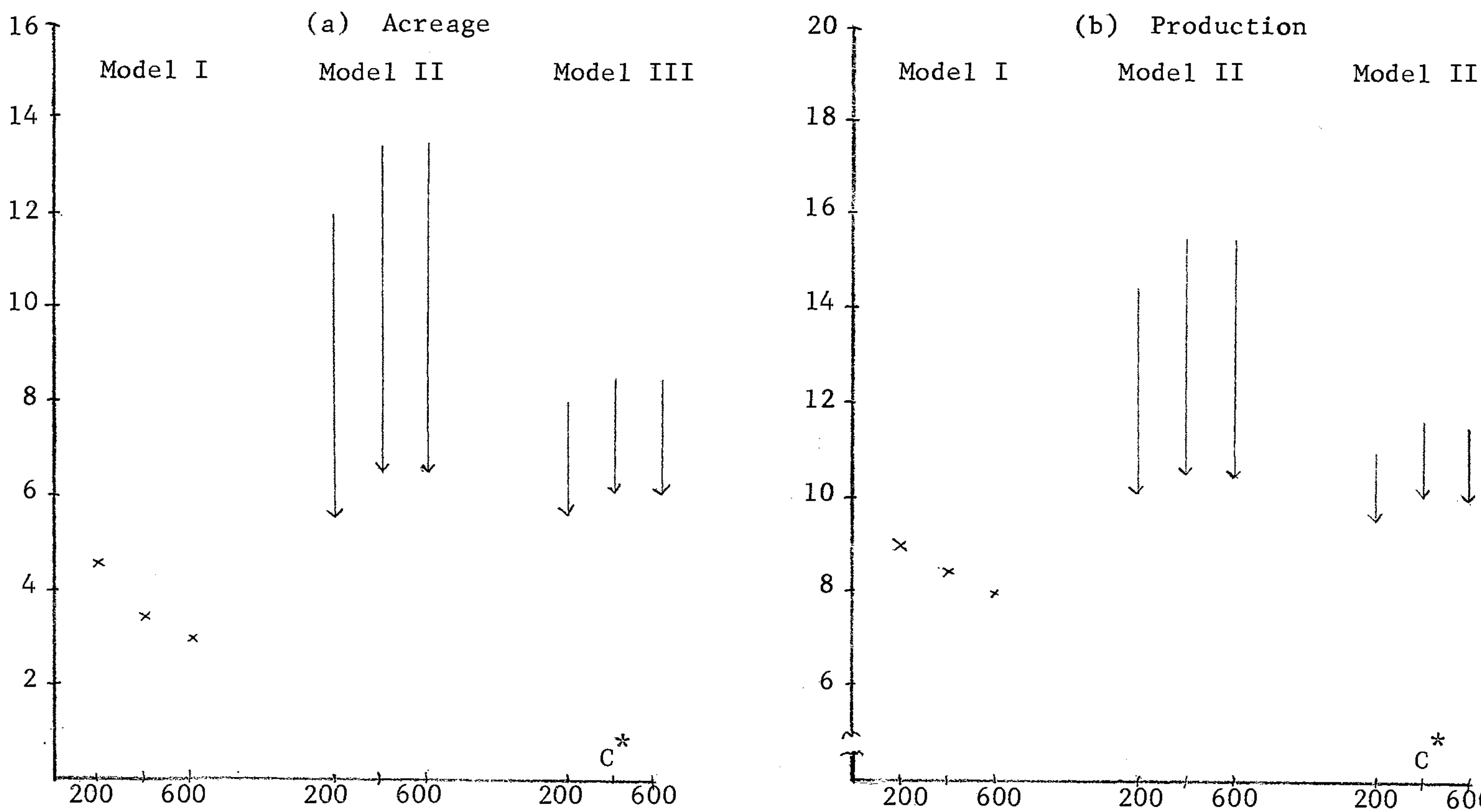


Figure 11. Ranges in Coefficients of Variation of Six Variables, Models I, II, and III

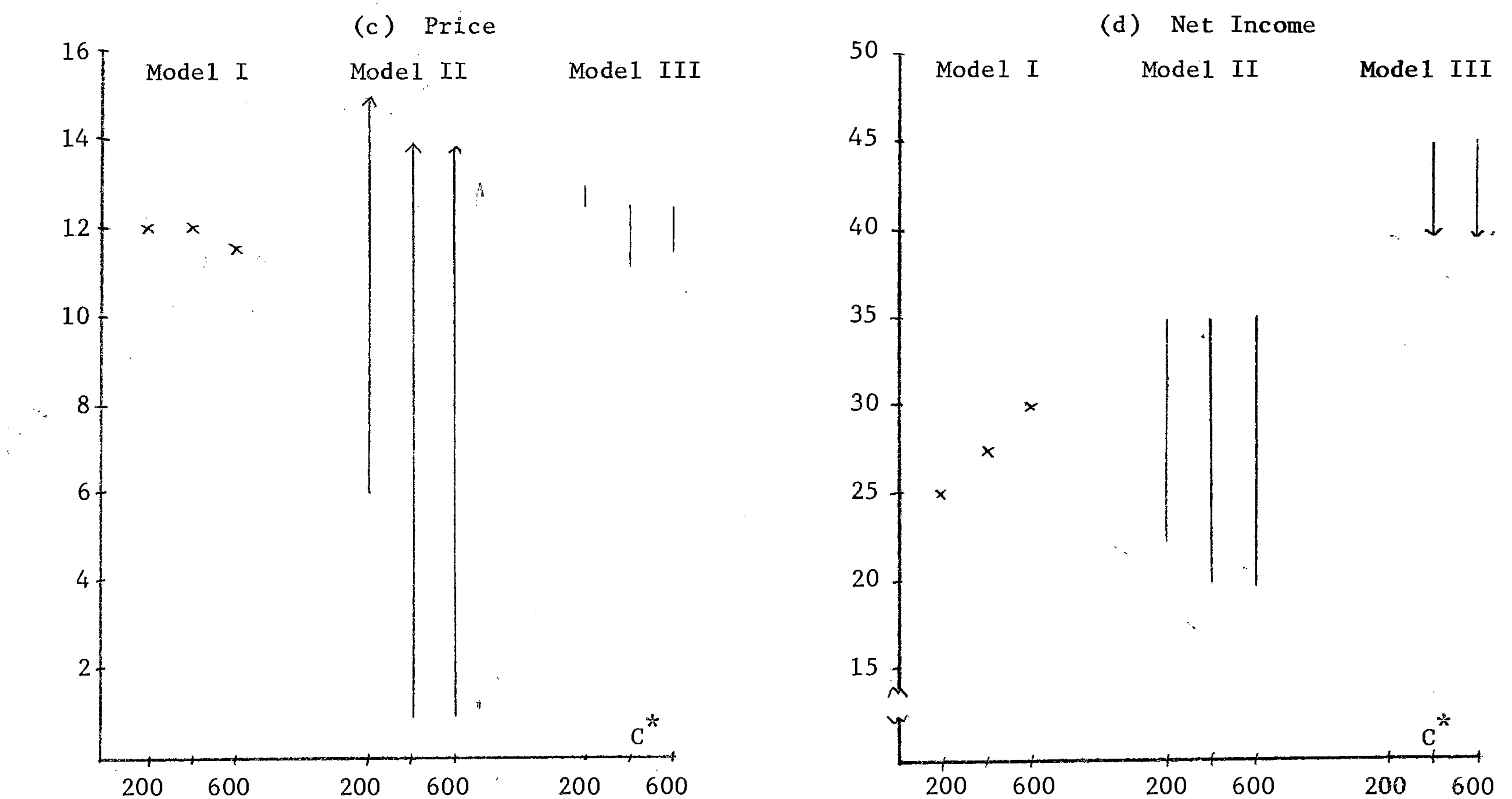


Figure 11. (Continued)

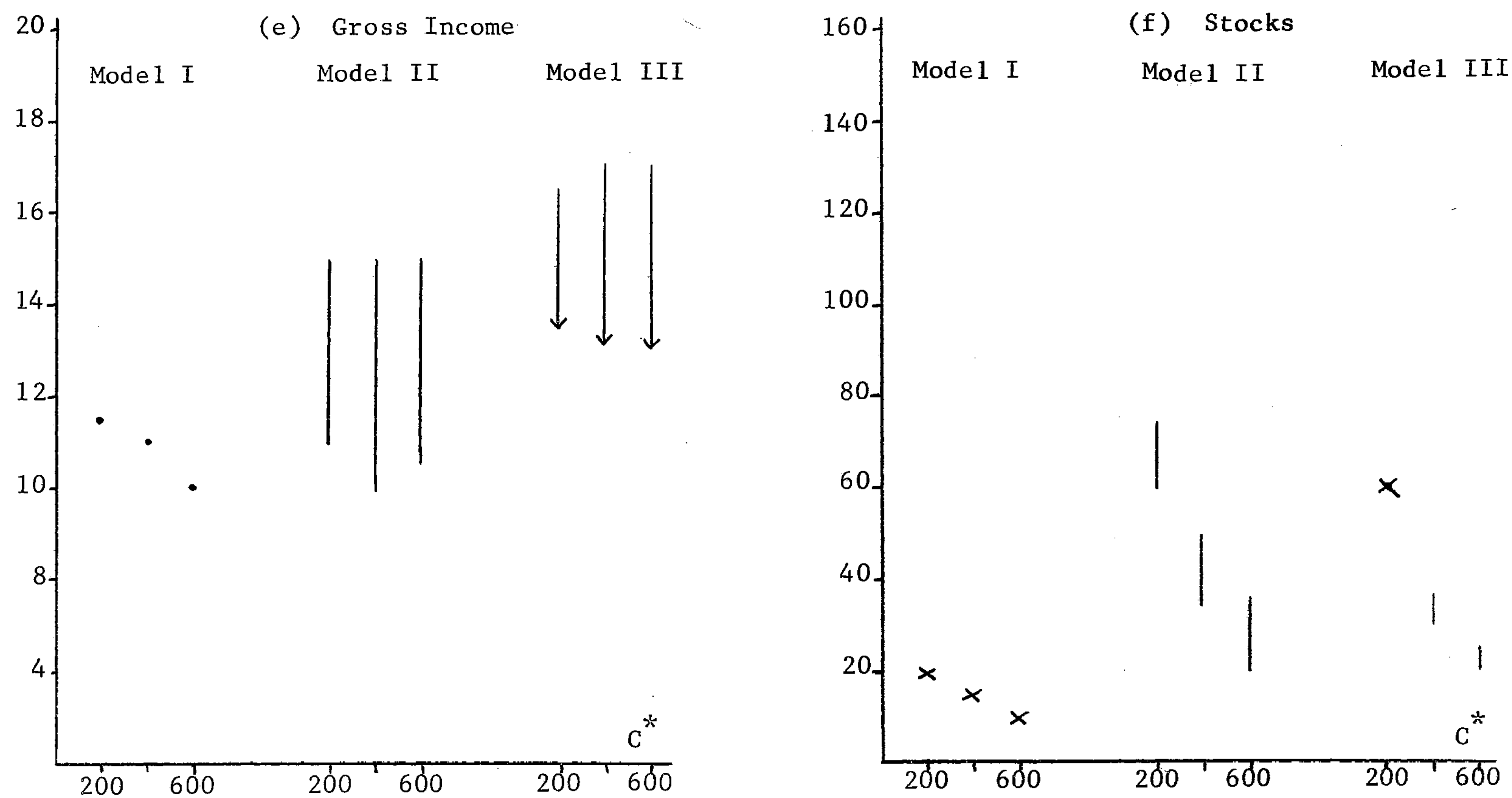


Figure 11. (Continued)

value of each variable shows as a single point. If a line is directional, it indicates that as the price spread widens (for Model II) or as θ becomes smaller (for Model III) the mean values show ever increasing or decreasing values in the direction of the arrow. Thus if a line points leftward, the mean value of that variable shows a constant trend toward a less desirable level as the price spread increases or as θ decreases, whichever is applicable.

Figure 11 presents the same type of information for the coefficients of variation. For this figure, each vertical line covers the range of coefficient of variation values for the variable and model indicated. A directional line downward indicates consistent movement toward greater stability as the price spread widens (for Model II) or as θ decreases (for Model III).

Summary

This chapter has presented selected results from simulating the reserve management policies represented by the three basic models discussed in the preceding chapter. Several variations were considered for two of the models, and summary measures were calculated for a number of variables. This section does not attempt to summarize the results from the many situations considered: a summary discussion of the results and implications of the simulation analysis is given in the final chapter.

It is apparent that the models do not react alike with respect to supply conditions. For example, when acreage is set each period at a level designed to achieve a certain carryover level, Models I and III are quite sensitive to the target carryover used, but Model II is not.

Also, whether acreage is tied to carryover or to price has more effect on the performance of the systems represented by Models I and II than for that represented by Models III.

CHAPTER VI

INTERNAL MODEL VARIATIONS

Introduction

All of the simulated situations studied so far have differed from the others only in either the demand for stock function or in the short-run supply function. The demand for stock function has taken on three different forms corresponding to three different reserve management policies, two of which were simulated with a number of variations. Two different short-run supply functions have been simulated including three variations for one of them. All other parameters and relationships within the model have remained fixed throughout the analysis.

It has been mentioned earlier that many of the parameter values and specifications used within the model were subjectively or, in some cases, arbitrarily chosen. This was often because of a lack of better information that could have led to more objective determination of such parameters and relationships. After the model was developed and the three models simulated, certain internal changes were again made as an attempt to provide additional information. The simulation model is flexible so that a change can be easily incorporated into the model.

Given that specification or observation errors exist within the model, some information as to the potential seriousness of these errors can be obtained by resimulating the system after assuming a new value or a series of new values for a parameter. In particular, three separate changes were made in the model parameters and relationships that were previously assumed fixed. Two of these changes are related and pertain to demand while the other pertains to supply. Numerous other changes could have been made.

The first two sections examines minor changes in the assumed functional relationship representing the demand for exports. One change assumes a new level of demand, the other a new slope while retaining the same level of demand at the 120 equilibrium price. It was decided to concentrate interest on the export demand function because this is the most volatile and hardest to objectively determine of all the demand components: there is definitely not substantial agreement among researchers and grain trade officials as to the proper representation of the U. S. wheat export demand. Also, this demand component is most likely to shift over time: a functional representation that holds true today is likely to be in error at any future time.

The third section looks at another acreage determination technique; namely, as if acreage is set at the assumed equilibrium value of 62 for each period. This change can be viewed in two ways. It represents a basic change in supply management philosophy and could be considered as a separate model instead of a sensitivity analysis. But it also allows the simulation results to be examined as if the acreage determination decision were not made internally or even based on values generated by the model. In this sense, it is a sensitivity analysis showing the reactions of the system with and without consideration of acreage decisions.

Only one model was simulated for each of the three changes. Each section of the chapter gives some possible generalizations to the other reserve management policies.

Change in the Level of Export Demand

This section describes the changes within the model and presents the results obtained by assuming a new level for export demand.

The only change actually made within the model moves the demand for exports curve far enough to the left so that the new equilibrium level for exports is 600. Since there are no offsetting shifts in other demand functions, this change has repercussions throughout the model. For example, a new aggregate demand function results which establishes new equilibrium values for all demand components since no changes are made in the supply situation.

The change necessary to accomplish the desired result is shown in equation (1) and the resulting new aggregate demand function is given in equation (2). New demand component equilibrium conditions and resulting elasticities at the equilibrium price are shown in Table XXVII.

$$QE_t = 518.36 - 3.3125P_t + .75QE_{t-1} + \varepsilon \tag{1}$$

$$Q_t = \begin{cases} 1213.36 - 3.5625P_t + .75QE_{t-1} + \varepsilon & P > 130 \\ 2383.36 - 12.5625P_t + .75QE_{t-1} + \varepsilon & P \leq 130. \end{cases} \tag{2}$$

The acreage determination function for the situation where acreage is assumed to be dependent on market conditions retains the same functional form and the same short- and long-run elasticities of .3 and 1.0 respectively.

TABLE XXVII

EQUILIBRIUM AND ELASTICITY VALUES, MODEL II'

Component	Equilibrium Level[a]	Elasticity	
		Short run	Long run
Price	111	--	
Food	567	- .049	
Feed	269	-3.72	
Export	600	- .61	-2.44
Total	1436	- .97	

[a]Units correspond to those given in footnote 1, Chapter IV.

The new equilibrium price of 111 shown in Table XXVII is 7.3 percent lower than the old equilibrium of 120. With unchanged food and feed demand functions, this 7.3 percent drop means that the increase in quantity taken at equilibrium is about .4 percent for food demand and 41 percent for feed demand, or absolute increases of 2 and 79 respectively. These increases together with the decrease in exports of 195 mean that total equilibrium demand is 114 million bushels or 7.3 percent less than before.

Only Model II under the five "equilibrium" situations was resimulated using the new conditions. The same four conditions of acreage determination were used. These situations are designated as II'_M, II'_2, II'_4, and II'_6 in the following presentation.

Table XXVIII shows the means and coefficients of variation for three situations of Model II' and the percentage changes of each from

TABLE XXVIII

SELECTED SIMULATION RESULTS: ABSOLUTE AND PERCENT CHANGE FROM MODEL II TO MODEL II'

Model	Variable	Situation 1[b]				Situation 10[b]				Situation 12[b]			
		Mean[a]	Percent Change	Coef. of Var.	Percent Change	Mean[a]	Percent Change	Coef. of Var.	Percent Change	Mean[a]	Percent Change	Coef. of Var.	Percent Change
II'_M	Acreage	55.6	-10.8	4.6	- 6.1	57.5	- 7.4	4.1	- 2.4	57.6	- 9.4	5.5	- 3.7
	Production	1438	- 7.7	9.3	- 5.1	1437	- 7.4	9.0	- 3.2	1441	- 9.3	9.7	- 2.0
	Food	567	0.4	0.6	-25.0	567	0.4	0.5	139.0	567	0.4	0.8	-19.0
	Feed	273	36.5	23.4	-17.9	270	39.2	30.1	- 27.8	278	24.1	42.1	-15.1
	Export	598	-24.6	23.4	32.2	599	-24.4	21.8	23.4	595	-23.9	19.7	25.5
	Stocks	514	- 8.1	70.8	15.7	549	- 3.0	59.5	- 0.3	93	17.7	138.5	- 1.4
	Price	111.4	- 7.6	9.1	-15.8	111.3	- 7.4	8.9	- 9.3	111.6	- 9.2	14.5	-18.5
	Net Income	449	-28.6	36.0	4.3	443	-29.1	29.0	10.3	441	-30.3	39.5	- 3.7
	Social Cost	16.2	-24.0	165.9	-16.9	13.9	-12.0	152.1	- 21.1	29.1	-29.2	145.1	- 8.9
	Total Loss	93.4	-12.0	63.2	9.9	96.3	- 4.5	55.0	- 3.7	43.2	-18.5	105.7	-14.5
II'_2	Acreage	56.7	- 5.5	11.9	- 3.3	57.3	- 5.0	9.3	- 5.1	59.8	- 9.2	6.4	14.3
	Production	1415	- 4.3	14.1	- 3.4	1431	- 6.1	12.2	- 4.7	1494	- 9.3	10.0	0.0
	Food	567	0.4	0.20	-33.3	567	0.4	.42	- 20.6	568	0.5	.68	-12.8
	Feed	259	41.5	12.2	7.1	265	43.2	28.6	- 27.8	293	20.0	41.1	-10.1
	Export	590	-21.8	25.2	16.7	599	-21.1	23.1	- 15.5	633	-22.7	19.0	21.0
	Stocks	219	-11.7	77.0	4.0	203	-16.2	65.8	8.0	140	14.7	68.1	-12.7
	Price	112.6	- 7.4	4.9	-16.9	111.9	- 8.7	8.4	- 1.2	109.3	- 6.4	13.7	9.1
	Net Income	460	-26.5	29.6	6.8	447	-27.2	26.3	14.3	422	-27.5	38.8	9.6
	Social Cost	17.2	-28.9	132.1	- 3.9	20.0	-26.5	135.1	- 1.6	35.8	-26.5	134.0	- 2.1
	Total Loss	50.0	-18.8	78.8	4.0	50.6	-21.5	68.1	- 7.6	56.8	-10.8	83.0	- 8.8
II'_4	Acreage	57.7	- 5.4	13.5	7.4	578	- 4.9	10.4	- 1.0	60.2	- 8.5	7.2	12.5
	Production	1440	- 5.6	15.4	- 2.5	1443	- 5.1	13.0	- 3.0	1505	- 9.6	10.6	1.9
	Food	567	0.4	.14	27.3	567	0.4	.37	- 2.7	568	0.4	.65	-11.0
	Feed	267	42.0	5.0	0.0	269	43.8	27.2	- 31.0	296	18.9	40.9	-10.0
	Export	605	-21.6	26.5	16.7	607	-21.2	24.1	16.4	641	-23.0	19.8	23.7
	Stocks	394	- 7.3	49.4	1.0	391	- 9.1	38.5	2.9	329	7.9	33.0	- 4.4
	Price	111.4	- 7.3	1.4	16.7	111.2	- 7.7	7.3	7.4	108.7	- 6.1	13.4	- 5.0
	Net Income	450	-26.7	30.0	11.1	441	-27.3	24.7	17.6	417	-27.0	38.4	13.3
	Social Cost	22.2	-15.6	118.4	- 0.6	22.5	-17.7	132.7	- 3.9	40.2	-19.9	143.7	9.1
	Total Loss	81.4	- 9.6	46.8	-13.7	81.2	-11.4	43.4	- 14.1	89.6	- 6.4	61.1	7.6
II'_6	Acreage	57.9	- 5.6	13.8	2.2	57.8	- 5.9	10.6	0.0	60.3	- 8.5	7.3	8.9
	Production	1446	- 5.7	15.7	0.0	1445	- 5.1	13.1	- 2.3	1506	- 8.7	10.6	9.5
	Food	567	0.4	.15	25.0	567	0.4	.36	- 5.3	568	0.4	.66	-10.8
	Feed	270	40.6	2.9	-52.5	269	43.8	26.9	- 31.9	297	18.8	40.8	-10.1
	Exports	609	-27.6	26.7	18.7	608	-21.0	24.3	17.4	641	-23.2	20.0	23.4
	Stocks	588	- 4.7	34.0	0.0	590	- 6.4	26.0	2.0	528	5.2	20.7	- 5.5
	Price	111.1	- 7.3	0.81	-26.4	111.2	- 7.0	7.2	5.9	108.7	- 6.0	13.4	- 5.0
	Net Income	449	-26.6	30.4	11.1	441	-27.3	24.7	17.6	416	-27.2	38.5	13.2
	Social Cost	23.9	- 7.7	129.4	12.2	23.1	-14.5	136.1	0.5	40.6	-21.9	143.9	6.7
	Total Loss	112.1	- 5.4	32.4	-10.5	111.6	- 9.2	31.8	- 17.0	119.8	- 5.9	46.1	11.9

[a]Units correspond to those given in footnote 1 of Chapter IV.

[b]Situation numbers correspond to those given in Table XI.

the same situations of Model II. The results from only three situations are presented in this table because it is obvious that, especially in the case of the means, the results under the new conditions are nearly always either almost the same or follow linear trends that hold for all situations. Given the results that hold for one or two situations, those from other situations can be quite accurately predicted.

One of the most interesting and significant results is the consistency and size of the change in average net income. The 7.3 percent price drop and quantity increase brought about a decrease in net income of from 29 to 27 percent -- 29 percent in the case of situation II'_M and 27 percent for II'_2, II'_4, and II'_6. This is a very sizable decrease, and has rather serious empirical implications -- to the extent that the model is accurate.

This says that if export demand decreases so that the quantity exported decreases by 195, net income drops about 87 cents for each bushel drop in exports even though 48 percent of the quantity decrease is offset by greater quantities used by other sectors, primarily as feed. In percentage terms, the drop in net income is greater than one-to-one -- a 24.5 percent drop in exports results in a 27 percent decrease in net income or, put another way, a 7.3 percent drop in total demand gives a 27 percent lower income. The decrease in gross income is less severe: the 7.3 percent decrease in total quantity taken reduces gross income by about 14 percent. These statements are summarized in Table XXIX which clearly shows the severity in income losses sustained by reduced quantities taken.

It should be pointed out that severity of the net income drop is not very sensitive to the value used to represent variable production costs. Experiments conducted while the model was in the development stage seem to indicate that a 50 percent error in choosing the cost of production per acre would result in a net income error of less than 20 percent.

TABLE XXIX

SELECTED CHANGES RESULTING FROM REDUCED EXPORT DEMAND

Component	Model II Level[a]	Model II' Level[a]	Absolute Change[a]	Percent Change
Exports	795	600	195	24.5
Price	120	111	9	7.3
Acreage	62	57.4	4.6	7.4
Total Usage	1550	1436	114	7.4
Net Income	620	450	170	27.0
Gross Income	1860	1660	260	14.0

[a]Units correspond to those given in footnote 1 of Chapter IV.

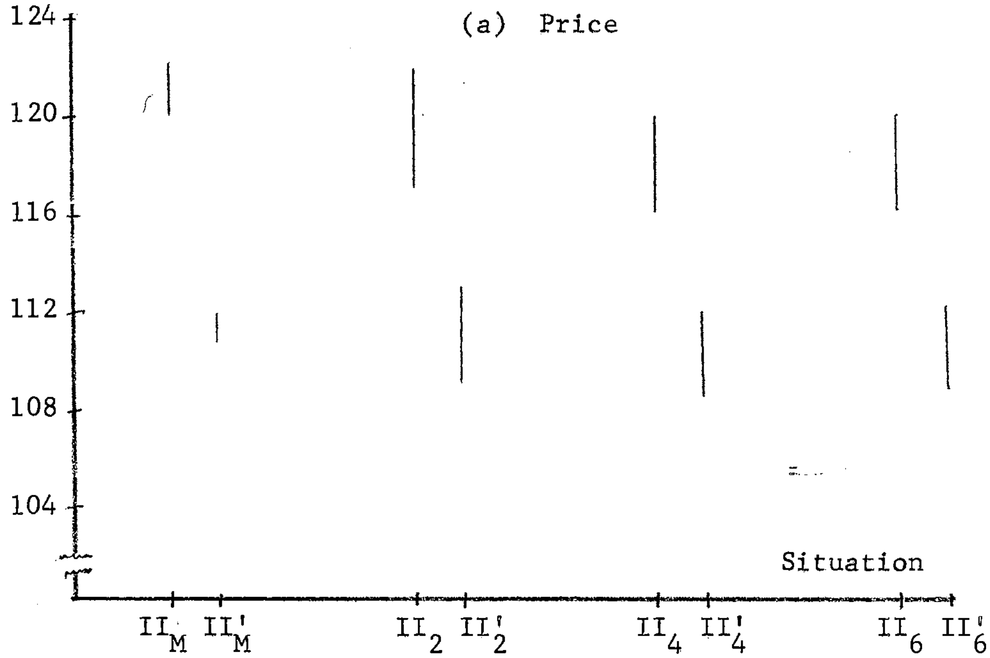
It is the reduction in absolute total demand that is tied directly to the reduced income values -- a decrease in any demand component has the same effects. However, only export demand is a large enough component of total demand to cause an absolute change of this magnitude. If another component were to drop a similar percentage, the results would be less severe for two reasons: 1) other components are relatively less important to total demand so that a similar percentage decrease would decrease total demand less in both absolute and percentage terms, and 2) larger offsetting increases in the unchanged demand components would result -- primarily because of the size and somewhat high elasticity of export demand.

About the only changes shown in Table XXVIII which are not strictly predictable or do not follow linear patterns are associated with situation 12, and these are explainable as likely random events. As explained earlier in the discussion of Model II, when the price spread is as wide as in situation 12, the market can operate without intervention for a wide range of production conditions so that it is very likely that a certain stock level may remain exactly at that level for many periods even though it is an undesirable or even an intolerable level. This means that average stocks for situation 12 are not really meaningful because a situation such as zero stocks is not likely to be actually tolerated for long.

Figure 12 illustrates the differences in mean values resulting from Models II and II'. The vertical lines cover the ranges in mean values resulting from the five situations of each model for the four supply determination conditions.

This figure together with Table XXVIII seems to indicate that the model is rather sensitive to changes in the demand quantity. The results also indicate that the model behaves in an orderly enough fashion so that proposed changes or new conditions can be analyzed without looking directly at all situations or supply conditions.

The figure and tables show that while the values of most key variables declined substantially, the ranges in mean values resulting from



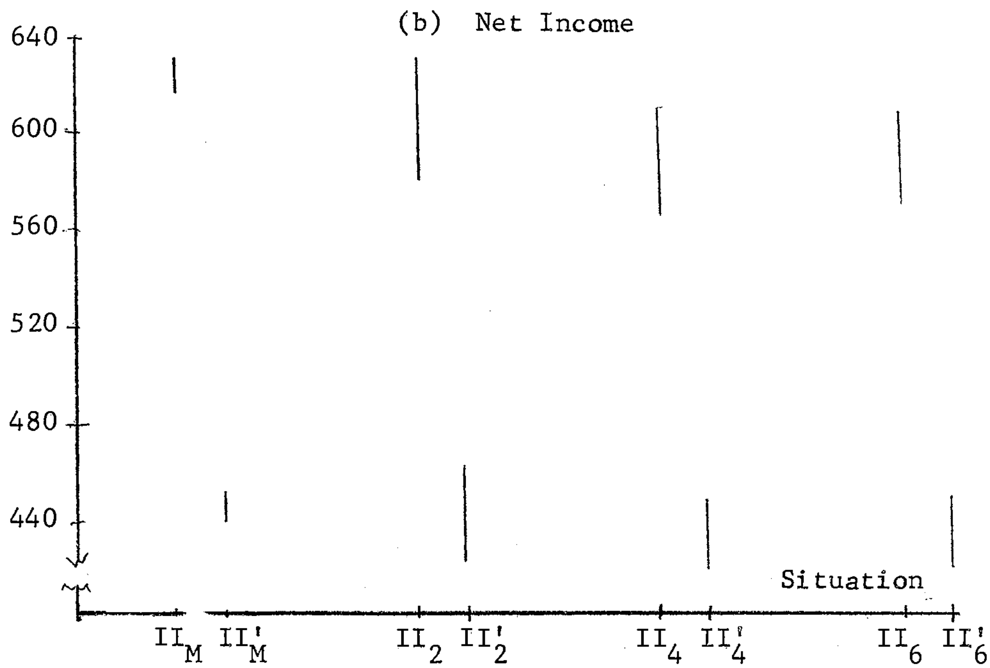
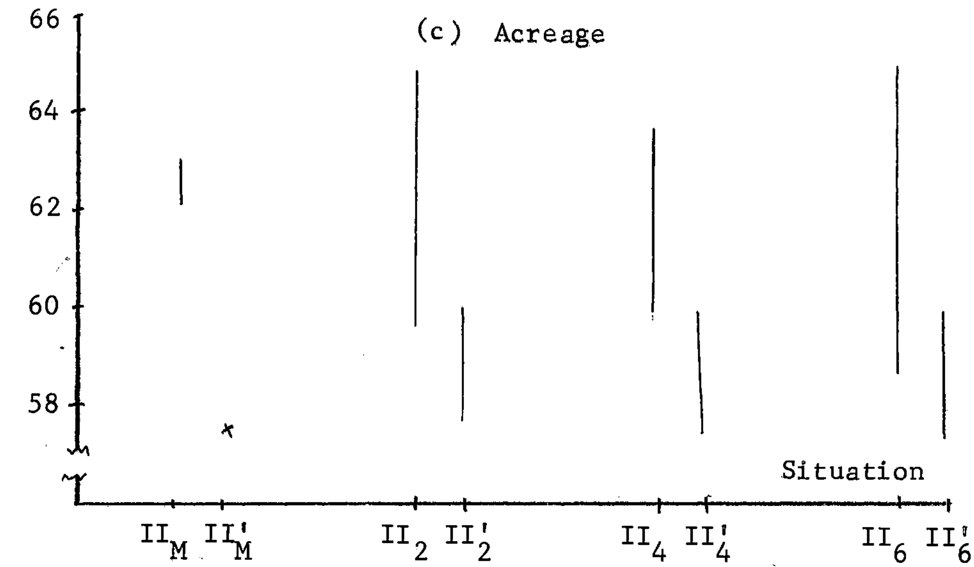


Figure 12. Ranges in Means of Six Variables, Models II and II'



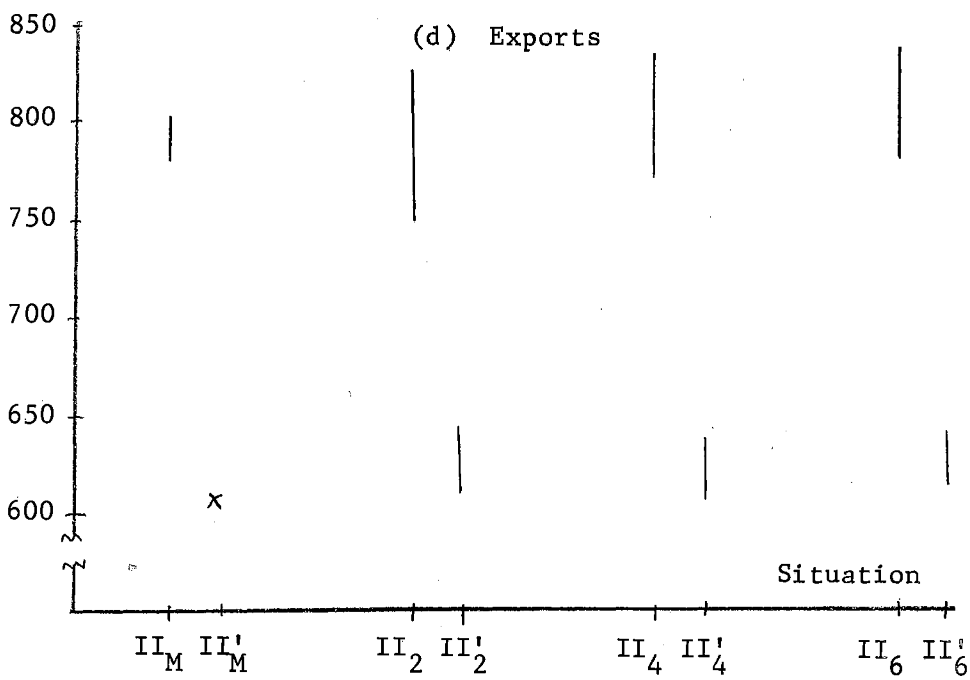
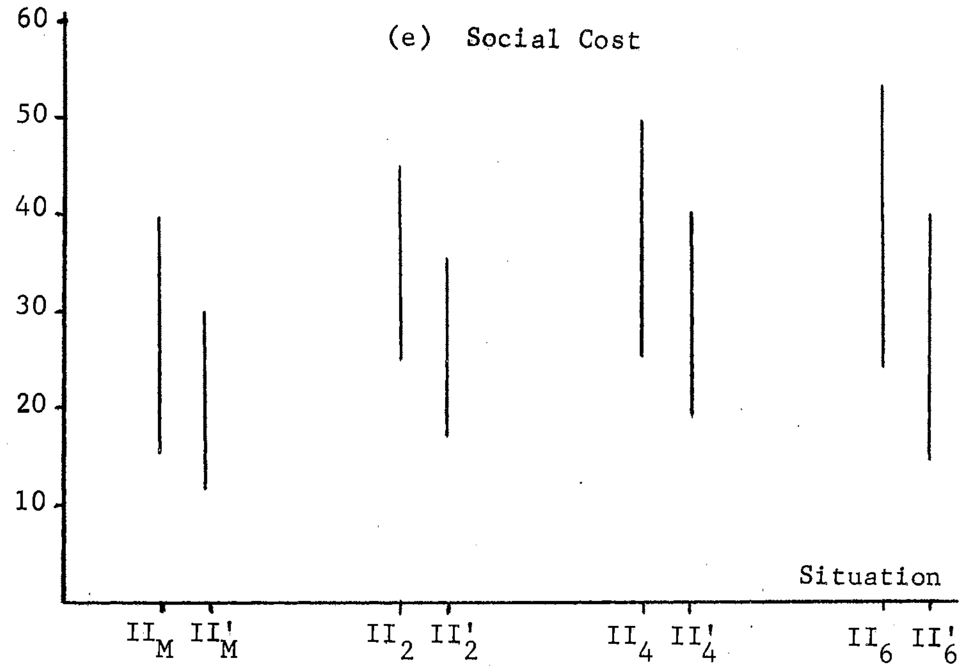


Figure 12. (Continued)



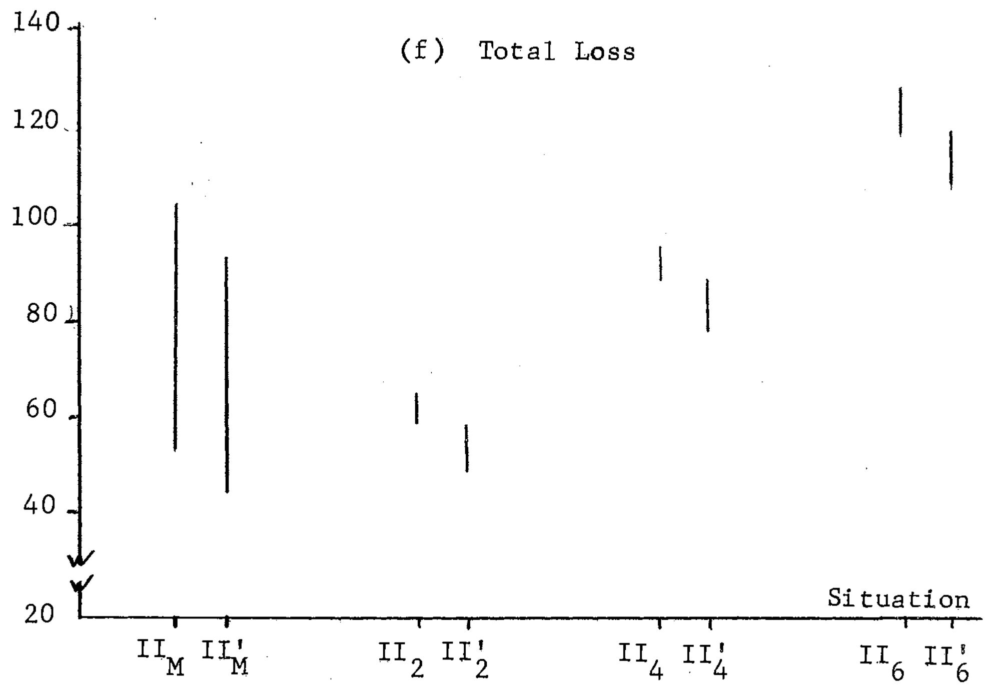


Figure 12. (Continued)

the three situations show only minor changes. The relative positions of the three situations also remain unchanged: relatively desirable or undesirable characteristics associated with a given situation in Model II show the same characteristics in Model II'. There is no evidence to show that the same would not hold true for this change applied to the other models. The relative sensitivity of all models to supply conditions should hold, and the desirable and undesirable features of each model should remain associated with the same model variations.

Change in Export Demand Elasticity

This section describes the changes made within the model and presents the results obtained by assuming a new slope parameter for the export demand curve while preserving all original equilibrium values. To accomplish this, a new location parameter or constant term is required as well as a new slope or price coefficient term. Again there are no offsetting shifts in the other demand functions so that another new aggregate demand function is formed.

The change made increases the export demand function's equilibrium short-run elasticity from -.5 to -2.0 and the long-run elasticity from -2.0 to -6.0. This change was made for two reasons: 1) to see how the model and the system react to a quite radical change in the elasticity of an important demand component, and 2) because it is quite possible that the previously assumed values are too low, and may in fact be as great as the new values.

The five situations of Model III using two supply determination considerations were selected to be resimulated using the new values. The acreage determination conditions where C^*, the target or desired carryover level is 200 and 600 were not resimulated. The two supply situations are designated as III'_M and III'_4 in the following presentation.

The export demand function necessary to give the desired elasticities is shown in equation (3) and the resulting aggregate demand function in equation (4).

$$QE_t = 1855 - 13.25P_t + .667QE_{t-1} + \epsilon, \tag{3}$$

$$Q_t = \begin{cases} 2250 - 13.50P_t + .667QE_{t-1} + \epsilon, & P > 130 \\ 3720 - 22.50P_t + .667QE_{t-1} + \epsilon, & P \leq 130. \end{cases} \tag{4}$$

The aggregate demand function has short-run price elasticity of -1.74 at the equilibrium price of 120. The 400 percent increase in short-run export demand elasticity gives a 90 percent increase in total demand elasticity -- from -.972 to -1.74.

Table XXX gives the means and coefficients of variation of nine variables of Models III'_M and III'_4 for all simulated situations as well as the comparable values from Models III_M and III_4. The differences between results can probably be best analyzed by looking first at the changes in stability between the two models as indicated by the coefficients of variation.

The increase in stability under the new conditions is quite striking in some cases. A given change in quantity marketed now causes price to change much less than before so that greater stability is induced throughout the system as the various elements interact dynamically.

TABLE XXX

SELECTED SIMULATION RESULTS, MODEL III'

		Means[a]				Coefficients of Variation			
		Situation							
Variable	θ	III'_M	III_M	III'_4	III_4	III'_M	III_M	III'_4	III_4
Acreage	1.00	--	--	62.0	62.0	--	--	8.2	8.3
	.90	62.0	62.4	63.6	63.6	2.1	6.4	7.3	8.3
	.80	62.0	62.6	65.2	65.2	2.1	5.7	6.5	6.8
	.75	62.0	62.0	66.0	66.0	2.1	5.5	6.1	6.4
	.70	62.0	62.5	66.8	66.8	2.1	5.8	5.7	6.1
Production	1.00	--	--	1550	1550	--	--	11.6	11.4
	.90	1552	1560	1590	1590	8.7	10.6	11.0	10.9
	.80	1552	1563	1630	1630	8.7	9.9	10.5	10.4
	.75	1552	1550	1651	1650	8.6	10.2	10.2	10.1
	.70	1552	1558	1670	1670	8.6	10.1	10.0	9.9
Price	1.00	--	--	120.1	121.3	--	--	4.7	12.5
	.90	120.0	120.8	119.2	119.2	5.1	13.5	4.7	12.0
	.80	120.0	121.0	118.4	117.2	5.3	13.5	4.8	11.8
	.75	120.0	120.0	118.0	116.2	5.4	14.0	4.9	11.8
	.70	120.0	121.0	117.6	115.2	5.5	14.3	5.0	11.9
Exports	1.00	--	--	796	780	--	--	6.1	11.6
	.90	797	784	828	807	8.2	13.9	6.0	11.6
	.80	796	784	861	833	9.2	14.5	6.0	11.6
	.75	796	784	877	840	9.5	14.8	6.1	11.6
	.70	796	784	893	860	9.7	14.3	6.2	11.6
Stocks	1.00	--	--	401	400	--	--	31.7	31.5
	.90	193	303	361	360	97.7	104.8	32.5	32.3
	.80	111	165	321	320	109.7	105.1	33.2	33.1
	.75	91	112	300	300	113.8	114.6	33.6	33.5
	.70	76	99	281	280	112.1	112.3	34.0	33.7
Net Income	1.00	--	--	622	640	--	--	29.3	44.9
	.90	621	634	624	622	25.7	41.7	28.1	42.9
	.80	620	635	626	603	24.1	38.3	26.9	41.5
	.75	620	610	626	593	23.4	37.5	26.3	41.0
	.70	625	625	626	584	22.8	37.6	25.3	40.7
Gross Income	1.00	--	--	1862	1880	--	--	12.5	17.0
	.90	1862	1882	1896	1893	9.1	16.0	11.6	15.5
	.80	1861	1887	1929	1907	8.6	19.2	10.6	14.2
	.75	1860	1850	1945	1913	8.3	13.5	10.1	13.7
	.70	1861	1875	1962	1919	8.1	13.5	9.2	13.2

TABLE XXX (Continued)

Variable	θ	Means[a]				Coefficients of Variation			
		Situation							
		III'_M	III_M	III'_4	III_4	III'_M	III_M	III'_4	III_4
Social Cost	1.00	--	--	0.5	2.6	--	--	101.7	158.8
	.90	5.2	11.8	2.7	4.8	245.6	273.5	95.7	107.6
	.80	7.6	14.4	9.7	11.9	194.1	270.7	78.1	94.1
	.75	8.6	16.7	14.9	17.4	179.8	225.6	75.5	89.5
	.70	9.6	18.8	21.4	24.0	168.4	202.4	74.2	86.2
Total Loss	1.00	--	--	60.6	62.6	--	--	31.4	31.2
	.90	34.2	57.2	56.9	58.8	85.0	97.8	33.6	33.4
	.80	24.3	39.1	57.8	59.9	93.7	116.1	38.9	40.3
	.75	22.2	33.5	60.0	62.4	97.8	124.7	42.4	44.7
	.70	20.9	33.6	63.5	66.0	101.4	123.3	46.1	49.2

[a]Units correspond to those given in footnote 1 of Chapter IV.

The increased stability for some variables is much more pronounced than for others, with the degree of change seemingly tied to how closely that particular variable is directly related to price. Export as a demand component is directly related to price and shows the greatest change, while stocks, not directly related under this inventory management policy, show very little change. Only when acreage is tied to price the previous period as in situation III'_M does the more stable price give more stable acreage.

Many of the mean values shown in Table XXX are unchanged. Those that are different are due to either the nature of the reserve management policy or to the increased stability. For example, the fact that price varies less is undoubtedly responsible for the lower social cost and total loss values. A side effect of this result is that for situation III'_M, social cost does not increase at an increasing rate as θ becomes smaller and the total loss function does not reach a minimum for the θ values used. The minimum total loss value for situation III'_4 is associated with the same θ value as for III_4. This indicates that when the acreage decision is based on market conditions, the slope of the demand curve does affect the shape of the total loss function and hence the optimum storage policy as judged by a total social loss criterion.

As was explained in the earlier discussion of Model III, this reserve management model results in a bias toward less than equilibrium prices and smaller than desired stocks when θ is less than one and when acreage is tied to the deviation of actual from desired carryover the previous period because total supply will be above 1550 more often than below. Now with a less steeply negative demand curve, the price bias is less than before because a given excess supply gives a higher price. The result is responsible for the greater income values as well as higher prices for these situations of Model III_4' than for III_4.

Overall, the results of Models III_M' and III_4' compared with III and III_4 seem to indicate that a sizable change in export demand elasticity caused no great changes in means of the important economic variables. But the change did have marked effects on the stability of many series. If in fact the true elasticity of demand is close to that assumed in this section, previously reported mean values are not likely to be seriously in error, but relative stabilities as indicated by the coefficient of variation measure are likely to be overstated for many variables.

Comparing these results to those of the previous section seems to indicate that an error that gives an incorrect equilibrium is more serious than one that gives the wrong shape to a demand relationship but has correct equilibrium values.

Change to Constant Acreage

This last section describes the results from assuming that acreage is fixed at the previously assumed equilibrium value of 62. As mentioned earlier, this represents a radical change in reserve management policy. Now all upsets to the system, such as unusual demand or yield conditions, must be handled entirely by price adjustments (and variables related to price, such as demand quantity), or by the system returning to normal.

This fixed acreage system was simulated using the reserve management policy of Model III with the -2.0 and -6.0 values for short- and long-run export demand elasticity as described in the preceding section. This new situation is designated Model''. Table XXXI gives four measures on eleven variables for Model III'' and for Models III'_M and III'_4 which differ from III'' only by the way in which acreage is determined.

Table XXXI shows that with the exception of social cost, the mean values resulting from the new condition are not much different from when acreage is determined according to market conditions as in III'_M. What other differences do exist are small enough so that they are very likely explainable as due to random events.[1]

There is a significant decrease throughout the system in stability as measured by the coefficients of variation -- significant at least in

[1]For this particular simulation, the computer program was constructed to generate the same random numbers for all five values and for all three acreage situations, but the number of iterations differ. Models III'_M and III'_4 were simulated for 2000 periods and III'' for only 500 periods so that the differences that show between III'' and III'_M could be random. For example, there is nothing in the model to cause price to be even 0.4 higher for III'' than for III'_M. If this difference is random, it could be because of differences in the series of random numbers generated for either yield or export demand. In fact, the average yield for III'' was .210 less than the expected value of 25 and .018 more for III'_M and III'_4. This is probably enough to cause the differences in mean values that exist.

TABLE XXXI

SELECTED SIMULATION RESULTS, MODEL III''

Variable	θ	Mean[a] III''	Mean[a] III'_M	Mean[a] III'_4	Coefficient of Variation III''	Coefficient of Variation III'_M	Coefficient of Variation III'_4	Minimum[a] III''	Minimum[a] III'_M	Minimum[a] III'_4	Maximum[a] III''	Maximum[a] III'_M	Maximum[a] III'_4
	1.00	1537	--	1550	9.4	--	11.6	1302	--	1078	1860	--	2158
	.90	1537	1552	1590	9.4	8.7	11.0	1302	1228	1130	1860	1924	2183
Production	.80	1537	1552	1630	9.4	8.7	10.5	1302	1222	1183	1860	1919	2206
	.75	1537	1552	1651	9.4	8.6	10.2	1302	1221	1210	1860	1916	2217
	.70	1537	1552	1670	9.4	8.6	10.2	1302	1220	1237	1860	1914	2228
	1.00	1911	--	1950	18.2	--	6.7	1302	--	1659	2066	--	2309
	.90	1088	1744	1951	14.1	13.4	6.9	1302	1225	1656	2368	2586	2314
Supply	.80	1636	1664	1951	12.6	11.1	7.0	1302	1225	1653	2171	2318	2317
	.75	1620	1643	1951	12.0	10.4	7.1	1302	1225	1652	2132	2266	2320
	.70	1608	1627	1951	11.6	10.0	7.2	1302	1225	1651	2096	2217	2321
	1.00	565	--	565	.07	--	.43	560	--	562	569	--	568
	.90	565	565	565	.20	.50	.43	560	560	562	569	569	568
Food	.80	565	565	565	.20	.55	.35	560	560	562	569	569	569
	.75	565	565	565	.20	.46	.48	560	560	562	569	569	569
	.70	565	565	565	.25	.50	.47	560	559	562	569	569	569
	1.00	188	--	189	29.0	--	25.2	100	--	100	326	--	294
	.90	190	190	196	29.5	26.4	24.4	100	100	100	326	323	309
Feed	.80	191	191	204	30.0	27.0	24.0	100	100	100	326	323	325
	.75	191	191	208	30.0	27.3	23.9	100	100	100	326	324	333
	.70	191	191	211	30.0	27.6	23.9	100	100	100	326	324	342

TABLE XXXI (Continued)

Variable	θ	Mean[a] III''	Mean[a] III'_M	Mean[a] III'_4	Coefficient of Variation III''	Coefficient of Variation III'_M	Coefficient of Variation III'_4	Minimum[a] III''	Minimum[a] III'_M	Minimum[a] III'_4	Maximum[a] III''	Maximum[a] III'_M	Maximum[a] III'_4
	1.00	779	--	796	8.7	--	6.1	544	--	688	886	--	888
	.90	778	797	828	9.8	8.2	6.0	541	536	717	922	960	946
Exports	.80	779	796	861	10.8	9.2	6.0	541	542	733	961	1008	997
	.75	779	796	877	11.2	9.5	6.1	541	543	740	977	1027	1023
	.70	779	796	893	11.6	9.7	6.2	541	543	747	991	1043	1049
	1.00	379	--	401	85.2	--	31.7	0	--	109	1116	--	760
	.90	154	193	361	118.2	97.7	32.5	0	0	95	736	932	687
Stocks	.80	101	111	321	130.8	109.7	33.2	0	0	83	497	615	614
	.75	85	91	300	134.3	113.8	33.6	0	0	77	437	537	577
	.70	72	76	281	136.6	112.1	34.0	0	0	70	382	466	539
	1.00	120.4	--	120.1	5.3	--	4.7	105	--	108	136	--	132
	.90	120.4	120.0	119.2	5.9	5.1	4.7	105	105	107	142	141	130
Price	.80	120.4	120.0	118.4	6.1	5.3	4.8	105	105	105	143	141	130
	.75	120.4	120.0	118.0	6.2	5.4	4.9	105	105	104	142	141	130
	.70	120.4	120.1	117.6	6.3	5.5	5.0	105	105	103	142	141	130
	1.00	610	--	622	31.7	--	29.3	214	--	176	1081	--	1217
	.90	608	621	624	29.6	25.7	28.1	224	207	190	1051	1124	1182
Net Income	.80	607	620	626	28.1	24.1	26.9	294	236	204	1026	1088	1143
	.75	606	620	626	27.4	23.4	26.3	303	243	210	1016	1072	1122
	.70	606	620	626	26.7	22.8	25.3	314	263	214	1007	1058	1100

TABLE XXXI (Continued)

Variable	θ	Mean[a]			Coefficient of Variation			Minimum[a]			Maximum[a]		
		III''	III'_M	III'_4	III''	III'_M	III'_4	III''	III'_M	III'_4	III''	III'_M	III'_4
Social Cost	1.00	4.1	--	0.5	306.8	--	101.7	0	--	0	73	--	2
	.90	8.6	5.2	2.7	223.2	245.6	95.7	0	0	0	103	123	15
	.80	11.4	7.6	9.7	178.6	194.1	78.1	0	0	0	100	120	45
	.75	12.7	8.6	14.9	165.7	179.8	75.5	0	0	0	99	120	66
	.70	14.0	9.6	21.4	156.8	168.4	74.2	0	0	0	99	120	91
Total Loss	1.00	61.0	--	60.6	75.2	--	31.4	0	--	16	168	--	114
	.90	31.7	34.2	51.9	93.8	85.0	33.6	0	0	14	122	161	114
	.80	26.6	24.3	57.8	101.2	93.7	38.9	0	0	13	100	129	133
	.75	25.4	22.2	60.0	103.8	97.8	42.4	0	0	12	99	120	149
	.70	24.8	20.9	63.5	106.1	101.4	46.1	0	0	11	99	120	169
Gross Income	1.00	1858	--	1862	10.5	--	12.5	1454	2321	1224	2321	--	2612
	.90	1848	1862	1896	9.7	9.1	11.6	1463	2291	1289	2291	2424	2582
	.80	1847	1861	1929	9.2	8.6	10.6	1534	2266	1358	2266	2374	2565
	.75	1846	1860	1945	9.0	8.3	10.1	1543	2256	1393	2256	2354	2554
	.70	1846	1860	1962	8.8	8.1	9.7	1554	2247	1429	2247	2338	2542

[a]Units correspond to those given in footnote 1 of Chapter IV.

the consistency of the change.[2]

This is the change one would expect to result from a constant acreage situation compared with one where acreage responds to price in the preceding period. In the latter case, acreage is able to adjust in period t to an unusual situation in period t-1 so that there is less likelihood of extremely wide variations in economic variables resulting from a run of unusual conditions. Although most of the differences in the coefficients of variation between the two models are not large in absolute terms, some are greater than 20 percent for the more volatile components such as export use and inventories.

The only markedly different mean value is for social cost -- about 50 percent greater for III'' than for III'_M. This increase is directly attributable to the greater variability -- larger deviations from equilibriums result so that social cost is greater. As with situation III'_M, social cost does not increase at an increasing rate as θ becomes smaller, and although the total loss function is nearly flat for θ

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[2]There are two exceptions to this consistency. Social cost shows less variability as measured by the coefficient of variation but shows more variability when the standard deviation is the measure. This is an example of the fact that neither is a perfect indicator of stability. The coefficient of variation has been used throughout most of this presentation because of the sometimes sizable differences in means. For this particular comparison, the social cost variable is the only one where the comparative stability between conditions III'' and III' differs with the two measures.

Food consumption is also shown as less stable for III'' than for III', but it is likely that neither are accurate in actual value. Because food use varies so little, calculation of the standard deviation is likely to give quite wrong answers due to the inability of the computer to accurately represent internally very small or very large numbers -- one of which it is called upon to do, depending on the calculation formula used. For a very stable series, these small absolute errors become large relatively and the results can be inaccurate.

less than .80, it does not reach a minimum. This is contrary to the dynamic programming results presented in Chapter III where the acreage decision was not a part of that model as it is not a part of this simulation model. This could be an indication that failure to consider demand as a stochastic or dynamic variable in a dynamic programming model may bias the results. That is, since the failure of Model III' to give the same results as the dynamic programming model cannot be attributed to a difference in consideration of the acreage decision, it is possible that the difference lies in the way demand is handled. Demand was assumed stationary in the dynamic programming model used for this study, but is a stochastic variable in the simulation analysis.

Summary

Results presented in this chapter have several empirical and methodological implications. To the extent that the simulation model used here is a satisfactory representation of the real system, the results indicate that: 1) a shift in demand represented by a horizontal movement of the demand curve to the left markedly affects economic values of interest to all groups, 2) a change in demand represented by increased demand elasticity gives a more stable system, and 3) if acreage were the same each year, the system would be less stable.

The results show that some measures used in this study are sensitive to internal parameter values and that the nature of the sensitivity depends on the type of parameter involved. A change of the first type affects nearly all values in the same manner so that those situations showing desirable properties still do so. The second type of change is somewhat more selective than the first in that the increased stability also lowers social cost and total social loss. This change would bring about selecting a different value for Θ as being best when total social loss is the criterion and when acreage is market determined. A change of the third type is similar to the second in that the total social loss function is not U-shaped for the range of Θ values considered. However, a subjective evaluation of all measures indicates that those decision rules or situations previously reported as most favorable still retain these overall characteristics.

CHAPTER VII

SUMMARY AND CONCLUSIONS

The overall objective of this study is the analysis of U. S. wheat reserve management policies. Past and present government involvement in programs relating to reserve stocks indicate the need for more knowledge of the manner in which such programs affect certain economic variables.

Procedure

The study is composed of two separate but complementary analyses. The first part is devoted to determining an optimal wheat carryover policy that minimizes, via the dynamic programming technique, an hypothesized social loss objective function. The second part extends the results of the dynamic programming analysis with a simulation model of the wheat economy. This model is used to examine and compare several reserve management policies including that developed from the dynamic programming analysis.

The annual, aggregate models developed for these analyses do not deal with several questions that also need attention. Included in this category are: 1) optimal timing of inventory adjustments -- analysis requiring a model for which the effective period is shorter-than-annual, 2) optimal location for publicly controlled stocks -- analysis requiring a spatial model, 3) optimal carryover by type of wheat -- analysis requiring less aggregation, 4) distribution of costs and benefits among various interest groups -- analysis also requiring disaggregation but of a different type, and 5) least-cost distribution of government versus privately owned stocks.

Results and Implications

This section briefly summarizes the results of the dynamic programming and simulation analyses and attempts to indicate important methodological and empirical implications of the study.

The Dynamic Programming Analysis

Chapters II and III show that it is possible to formulate an aggregate wheat inventory model as a serial, multistage decision process suitable for optimization using the technique of dynamic programming. The key characteristic that makes this approach suitable is that an optimal decision regarding carryover this period requires that the distribution of carryover among all future years' supplies also be optimal.

Restrictions placed on the decision model to facilitate numerical optimization included: 1) the stage return values (net social loss values, developed from the hypothesized objective function) are assumed to be nondynamic and nonstochastic: the net social loss associated with each particular carryover level for a particular supply level is known and not dependent on either the period or on loss values for any preceding periods, and 2) stationary transition probabilities: the probability of a given supply for a particular period is dependent only on carryover from the preceding period and a random production variable which has the same probability density function for all periods.

The output of the dynamic programming optimization is a conditional decision rule showing the optimal amount of wheat to store each period for each possible supply amount. The results indicate that discounted net social loss over an infinite planning period is minimized if the quantity carried over into the next period is approximately 85 percent of the amount by which wheat production plus carryover exceeds the expected equilibrium demand quantity.

The Simulation Analysis

The simulation analysis extends the dynamic programming analysis in two directions. First, the model itself is refined by including 1) acreage decisions as an integral part of the model so that the random yield is not the only determinant of production, and 2) dynamic and stochastic demand elements. This is accomplished by including a lagged value and a random value in the export demand function. Secondly, the analysis is extended by examining reserve policies other than that from the dynamic programming results.

Two different types of acreage decisions are considered. One assumes that acreage follows a cobweb type pattern which can be expressed with a distributed lag equation. The equation assumes acreage is determined by purely market forces. The other assumes acreage is set semiexogeneously at a level designed to cover expected demand quantities plus a target carryover.

Three basic models of reserve management policy were developed and each simulated under a number of demand and supply conditions. The models represent quite different reserve management philosophies and require different methods of implementation.

Changing from present policy to that represented by Model I requires that the Commodity Credit Corporation no longer deal in stocks -- at least not in those stocks held for farm policy purposes of achieving income and price target levels or stability. Some agency such as the CCC could continue to purchase quantities for foreign and domestic food aid programs according to clearly stated rules of procedure and in known amounts. As such, these quantities would be a part of normal demand and would not affect inventory levels or management which Model I assumes is left to private interests within the grain trade industry. Price and income policy goals could still be achieved through government run production controls and direct payments, but not through purchases and sales of wheat stocks.

Models II and III do require public intervention on the demand side of the market. Model II uses price to indicate situations where a public agency such as the CCC, acting according to previously set and publicized rules, is required to enter the market to buy or sell stocks in sufficient quantities so that proper price relationships are maintained. The relative sizes of supply and demand quantities are not important in the operation of Model II until price indicates a situation where demand intervention -- purchase or sale of publicly controlled stocks -- is required to keep wheat prices within adopted, known limits. As with Model I, food aid purchases would be a component of normal demand, and operation of the inventory policy need not be affected by methods, such as production controls and direct payments, designed to achieve other policy goals.

Implementing Model III would require a public agency to intervene whenever quantity relationships dictate. Reserve stock levels would necessarily be publicly controlled (but not necessarily publicly owned), with additions made to these stocks during years in which the total available supply exceeds that of the preceding year and withdrawals whenever supply is less than the preceding year. Withdrawals or additions could be made throughout the marketing year at established rates, and adjusted to reflect the most recent supply estimates available. As with Models I and II, methods of achieving other national policy goals could be implemented outside the operation of the reserve stock management program.

Following are brief summaries of the simulation analysis results presented in Chapters V and VI.

Model I. For inventory Model I, stocks are considered a component of total demand, inversely related to price. Four hundred million bushels will be carried over into the next period when the price is equal to the assumed equilibrium price of $1.20 per bushel. The simulation results show that the values of key economic variables are quite sensitive to the target carryover value for those situations in which acreage is assumed to be set to give a particular carryover level. When this target carryover is less than 400, acreage, production and gross income are low; while price, social cost and total social loss are high. The opposite relationships hold when the target carryover is more than 400, except that social cost and total social loss are still much higher than when the target carryover is set at 400.

If the reserve stock function is to be left to market forces, a target carryover of 400 million bushels, with acreage set to achieve that carryover level, appears to be a policy with several advantages based on measures used in this study.

Model II. Model II represents a managed inventory policy which uses price as a key to indicate whether reserve stock adjustments should be made. For the Model II situations which use a uniform price range around the assumed equilibrium price as a key, the method of supply determination has considerable effect on the simulated results. When acreage is determined by market forces, situation 10 (where the upper price limit is 130 and the lower price is 110) shows the lowest social cost but also the lowest net and gross income figures. Model II is not very sensitive to values chosen for C^*, the target carryover value, for those situations where acreage is set exogenously. The income and social cost figures are generally less favorable, net income is more stable, and gross income and production conditions less stable for the controlled acreage situations than for the market determined acreage situations.

When the range around P^* is not uniform -- when the upper price limit (stock-selling price) is further above the equilibrium price than the lower price limit (stock-buying price) is below the desired or equilibrium price -- there are biases toward large stocks and toward high prices. The stock bias is limited when acreage is tied to the carryover level, and the bias towards high prices is not severe when acreage is tied to price. The results are that production, stocks and gross income are higher in the latter case; while price, net income and social cost are higher for the former. As long as the price range is not too severely one-sided, the income, social cost, and total loss values and stability conditions compare favorably with those occurring when the price range is uniform.

If an inventory policy of the type represented by Model II is selected, a subjective evaluation of measures used in this study indicates that the most favorable overall results would be obtained by 1) setting acreage each year to achieve a 400 million bushel carryover, and 2) setting the upper and lower price limits ten cents per bushel above and below the equilibrium price (II_4, situation 10).

Model III. Model III represents a managed inventory policy in which the excess of total available supply -- production the current period plus carryover from the preceding period -- over an assumed equilibrium quantity of 1550 million bushels determines the carryover level. The relationship between the size of succeeding periods' supplies determines the size of the inventory adjustment.

The results show that most summary measures are at least slightly sensitive to the value chosen for θ, the fraction of excess supply which is to be treated as carryover, and quite sensitive to the value of C^*, the desired carryover for those situations where acreage is tied to the size of the deviation of actual from desired carryover. A large value of θ gives comparatively high net income and large stocks but low gross income, low social cost and generally less stable conditions. The larger is C^*, the higher are gross income, social cost, total social loss and stock levels, but the lower is net income. Tying acreage to price gives greater gross but smaller net income than when acreage is tied to carryover.

Model III was designed to test the inventory policy approximated from the dynamic programming analysis which indicated that a θ value of .85 gives the lowest total social loss. The simulation results show this to hold for the controlled acreage situations. But $\theta = .75$ appears to be more nearly optimal for the market acreage situation. The results show that both social cost and total social loss are affected more by the supply determination condition than by the choice of θ.

A reserve management policy of the type represented by Model III would require that acreage be set outside the market to prevent an intolerably high probability of no reserves or zero inventory. Within the managed supply situations reported, a value of .80 for θ with a target carryover of 400 million bushels has several advantages based on measures used in this study.

Model Comparisons. It is difficult, for several reasons, to judge overall performance of all models. First, no reserve management policy rates consistently best for all conditions and criteria. Secondly, there is no criterion without obvious weaknesses or that will suit all interest groups. Also, the magnitude of the potential losses or gains from choosing one policy over another may be less than from the choices available within a single policy.

The various differences among and within models are seen most clearly and simply by referring to the graphic summaries of the three models in Figures 10 and 11.

Internal Model Variations. Chapter VI examines the simulation results from assuming: 1) a smaller export demand "location" parameter, 2) greater export demand elasticities, and 3) that the acreage decision not be a part of the system. These changes were designed to test the flexibility and sensitivity of the model as well as to provide information about the values of economic variables if these different assumptions are in fact true.

The first change reduces total demand and markedly affects the simulated wheat economy. The decreases in net income, social cost and total loss are considerably greater, in percentage terms, than is the decrease in quantity produced and sold. This implies that all markets are important to producers, particularly the export demand market because of its size relative to other demand components. The operation of the reserve management policy simulated did not seem to be affected by this change.

The most significant result of the second change is the increase in stability coming because of the greater export and total demand elasticities. The average values for most variables show only minor changes, but the range and coefficients of variation are noticeably smaller. Social cost and total social loss are markedly less as a consequence of this increased stability. In the market-determined acreage case, total social loss as a function of θ is not U-shaped as before for the values of θ which were used. This is an indication that the shape of the demand curve can affect the choice of an optimum storage policy as judged by a total social loss criterion. However, it should be noted that for θ values less than .80, total social loss remains nearly constant, falling only slightly.

Results from the final change indicate that if acreage is constant each period, price and income are more variable. Results also indicate that whether or not the acreage decision is a part of the model can influence the choice of a reserve management policy.

These summaries again show the difficulty of selecting any model as best -- each have a number of advantages and disadvantages. Model I has the obvious advantage that it requires no government intervention, but it does sacrifice desirable social cost and total loss features available by proper selection of situations of Models II or III.[1] Also, satisfactory operation of Model I when supply is set outside the market requires that the target carryover be very close to the amount that those performing the stocking function are willing to take at normal or equilibrium prices.

The reserve management policy represented by Model II would be relatively easy to operate. Interested persons could be readily informed about the type and estimated size of inventory adjustments forthcoming. This policy also has the advantage that stocks are somewhat insulated from the market, with the degree of insulation depending on the price spread used. Model II has the disadvantage that if the "correct" price spread is not selected, desirable income and social cost features will be sacrificed -- compared with what is available with other policies.

Advantages of Model III include low social cost and total social loss values and the fact that the penalties from choosing an incorrect θ value are not extreme. But this policy could not be used in the form given here with acreage market-determined because of the intolerably high likelihood of zero inventories.

From an overall point of view, the managed supply conditions seem to provide some advantages compared with leaving the acreage decision to the free market; the former gives generally lower social cost and total social loss, more stable conditions and often higher income values

[1] Although it can be shown that the free market will minimize social cost under static, equilibrium conditions (19, 20), there is no theoretical proof that this is true when supply or demand are stochastic.

without great sacrifices in any of the measures reported.

Also from an overall point of view, a reserve management policy of the type represented by Model II seems to be preferred to Models I and III. Model II represents a compromise between income and total loss and also shows generally more stable results. Model II also provides a degree of insulation from normal market operations, would be relatively easy to implement and operate, and would be relatively safe from completely depleted reserve stocks. Combining the overall evaluations given in this chapter and elsewhere would point to choosing a reserve management policy of the type represented by Model II with the acreage decision made to achieve a 400 million bushel carryover and with price limits ten cents above and below the equilibrium price.

Limitations and Suggestions for Further Research

The analyses presented here are restricted or limited in some degree to facilitate development of a basic methodology and to determine the feasibility of these types of analyses. Several of the restrictions and limitations have been discussed or alluded to in preceding pages, and many of them could be removed through additional research.

A very basic need is reliable information from which to develop parameters and input data for both the dynamic programming and simulation models. The results presented indicate that demand and supply location, slope and random variable distribution parameters need to be as accurate and as thoroughly tested as possible. Additional data are also needed in order to develop the spatial, less-than annual and disaggregated models mentioned earlier in this chapter.

Further information is needed to determine the relationships between working stocks -- those quantities in the industry pipeline that are about to be used or processed -- and inventories available as reserves. This study assumes that working stocks are a constant amount at all times and therefore have no effects on the analysis. This may not be the case -- the inventory level necessary for working stocks might vary seasonally or with total supply or size of inventory.

Further analysis could be directed toward developing a more sophisticated or realistic multistage inventory model. A more realistic representation of the system would be provided by formulating the problem into a two-decision-variable framework so that both carryover and acreage decisions would be optimal. Although difficult to confirm statistically, yield data exhibit some tendency toward runs of high or low average yields. Incorporating this feature into the model through nonstationary transition probabilities should improve the accuracy of the results as should methods to handle dynamic and/or stochastic stage return values.

The analysis could be extended to more completely include other related economic sectors. For example, the wheat economy is obviously tied to the feed grain and therefore the livestock economies. In this study, this relationship is manifest only in the feed grain demand function. In a dynamic framework, this is equivalent to assuming that feed grain demand and supply components move together with the wheat components so that both are in equilibrium at the same relative prices. Only in this way will the elasticity of the feed grain function correctly reflect alternative uses of wheat.

Finally, several refinements could be made in the reserve management policies studied and different policies could be considered. It seems reasonable to assume that as inventories become either very small or very large, a "good" reserve management policy would react differently than if the stocks were more nearly normal. Several "sliding-rule" refinements could be developed using the basic models of this study.

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APPENDIX A

TABLE XXXII

YIELD AND ACREAGE DATA USED TO DEVELOP EMPIRICAL PRODUCTION AND YIELD DISTRIBUTIONS

Year	Acreage	Yield/Seeded Acreage	Year	Acreage	Yield/Seeded Acreage
	million bu.	bushel		million bu.	bushel
1919	77,440	12.3	1944	66,190	16.0
1920	67,977	12.4	1945	69,196	16.0
1921	67,681	12.1	1946	71,518	17.1
1922	67,163	12.6	1947	78,314	17.4
1923	64,590	11.8	1948	73,345	16.5
1924	55,706	15.1	1949	83,905	13.9
1925	61,738	10.8	1950	71,287	14.3
1926	60,712	13.7	1951	78,524	12.6
1927	65,661	13.3	1952	78,645	16.6
1928	71,152	12.8	1953	78,931	14.9
1929	67,177	12.3	1954	62,539	15.7
1930	67,559	13.1	1955	58,246	16.7
1931	66,453	14.3	1956	60,655	16.6
1932	66,281	11.4	1957	49,843	19.2
1933	69,009	8.0	1958	56,017	26.0
1934	66,064	8.2	1959	56,706	19.7
1935	69,611	9.0	1960	54,906	24.7
1936	73,970	8.5	1961	55,707	22.1
1937	80,814	10.8	1962	49,274	22.2
1938	78,981	11.6	1963	53,364	21.5
1939	61,802	11.8	1964	55,672	23.1
1940	61,820	13.2	1965	57,361	22.9
1941	62,707	15.0	1966	54,513	24.1
1942	53,000	15.3	1967	67,595	22.5
1943	55,984	15.1			

Source: U. S. Department of Agriculture, Agricultural Statistics 1968. Washington: Economic Research Service, 1968, table 1.

APPENDIX B

Figure 13. Cumulative Means of Selected Variables Related to the Number of Simulation Iterations for Typical Situations

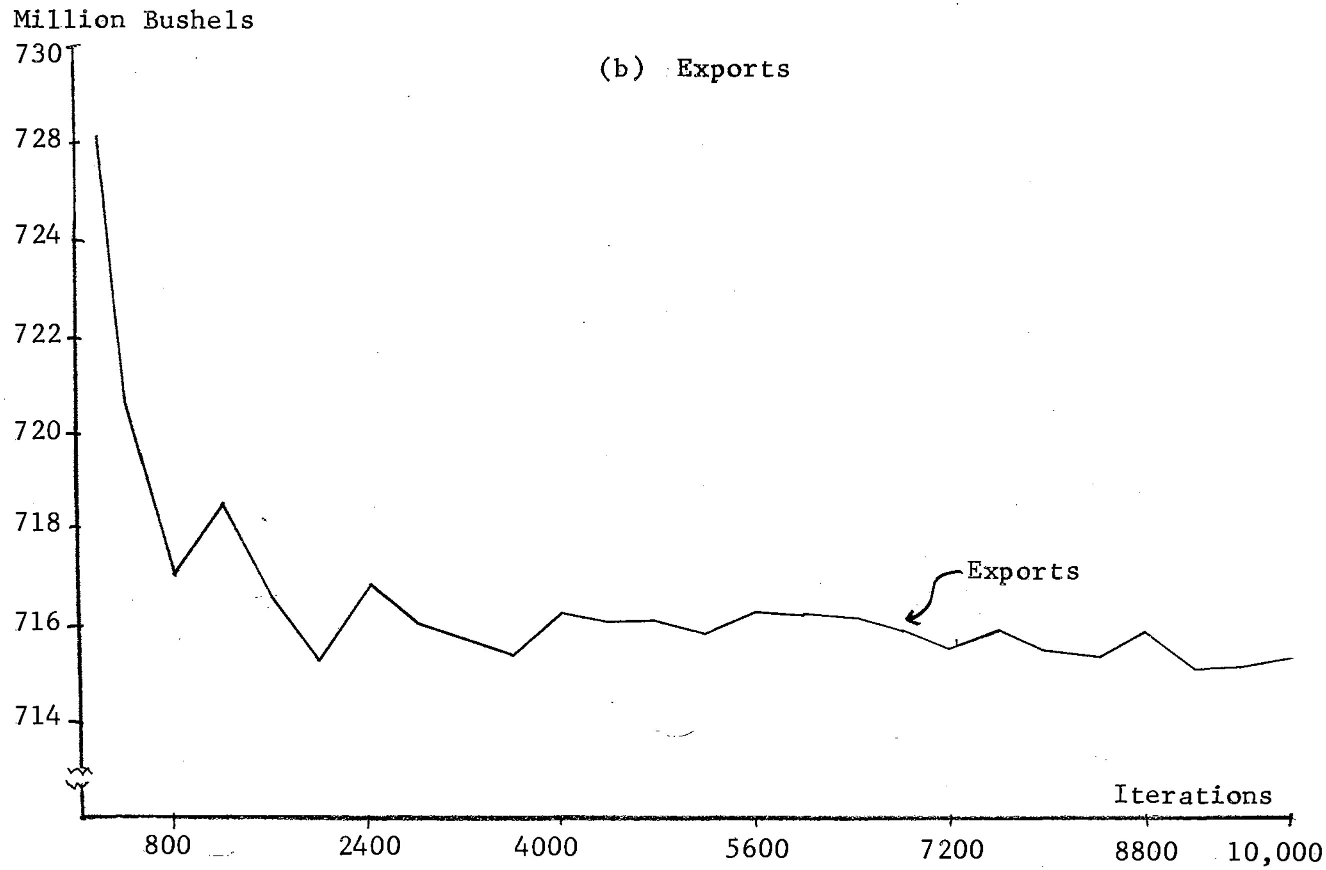

Figure 13. (Continued)

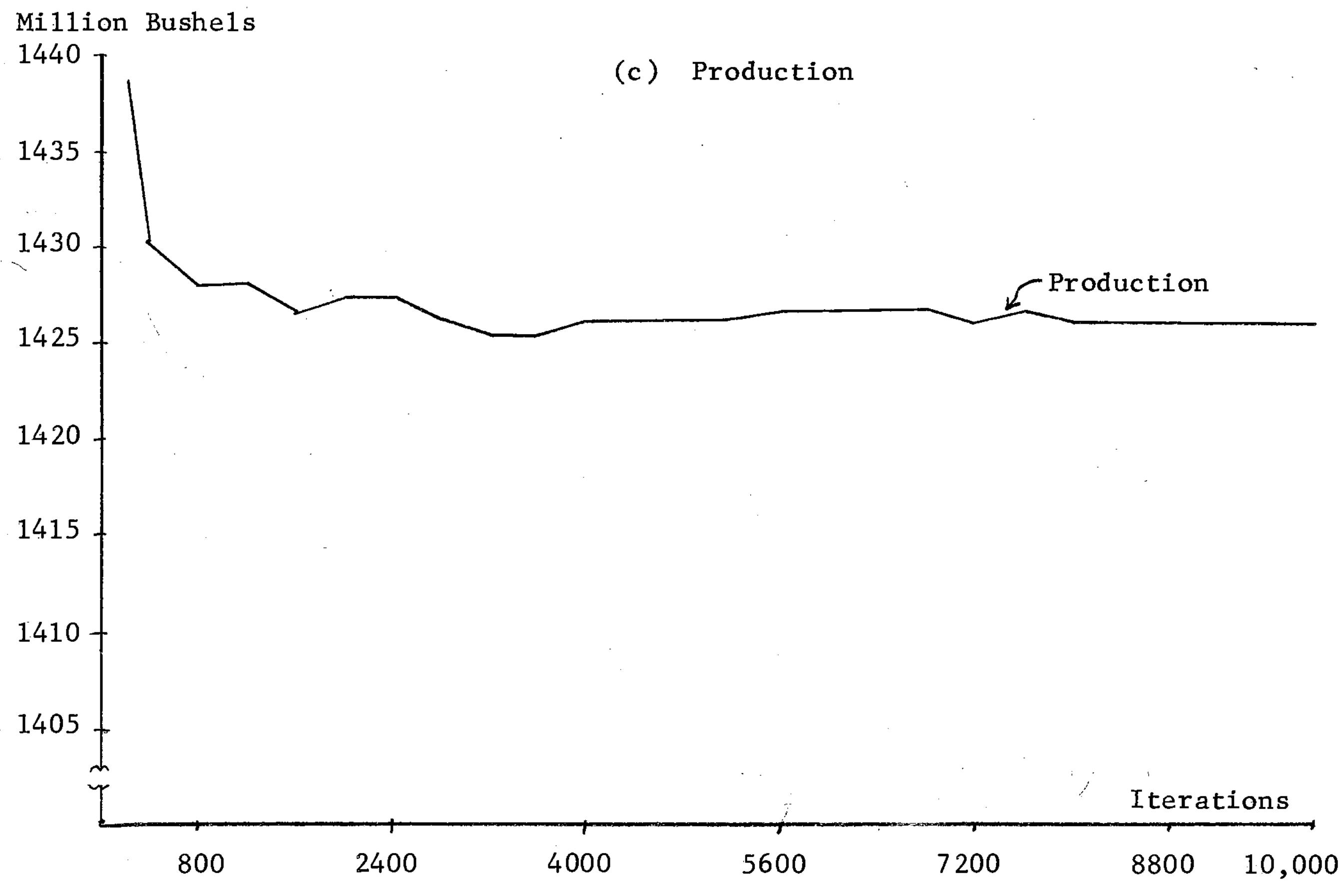

Figure 13. (Continued)

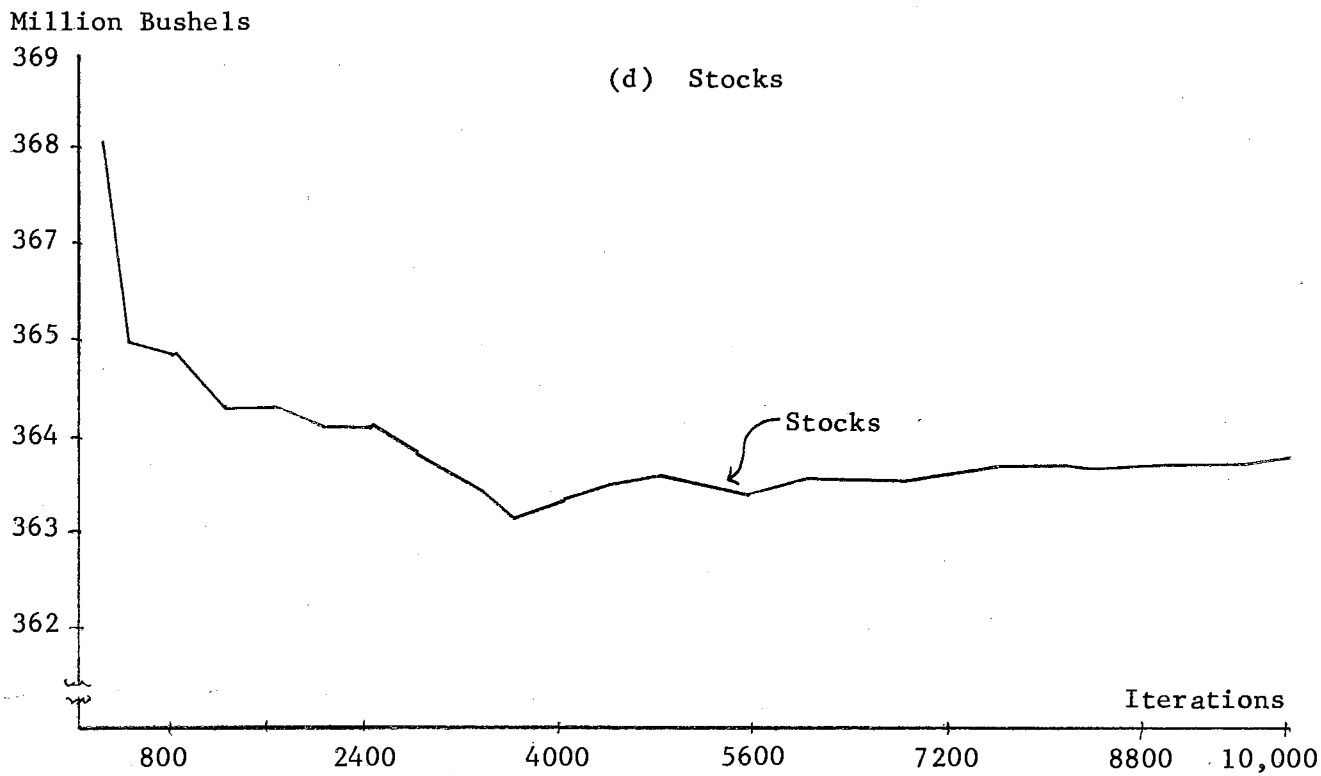

Figure 13. (Continued)

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